

# Homework 5, Topology

Due Tuesday November 25, 2008

Justify your answers.

1. Let  $\mathcal{T}$  and  $\mathcal{T}'$  be two topologies on a space  $X$ . Show that if  $X$  is compact and Hausdorff in both  $\mathcal{T}$  and  $\mathcal{T}'$ , then either  $\mathcal{T}$  and  $\mathcal{T}'$  are equal or they are not comparable.
2. Let  $\mathbb{R}_\ell$  be the real numbers with the lower limit topology. If  $[a, b] \subset \mathbb{R}_\ell$  has the subspace topology, is  $[a, b]$  compact?
3. Show that if  $h, h' : X \rightarrow Y$  are homotopic and  $k, k' : Y \rightarrow Z$  are homotopic, then  $k \circ h$  and  $k' \circ h'$  are homotopic.
4. Show that if  $U$  is an open connected subspace of  $\mathbb{R}^2$ , then  $U$  is path connected.
5. Given spaces  $X$  and  $Y$ , let  $[X, Y]$  denote the set of homotopy classes of maps of  $X$  into  $Y$ .
  - (a) Let  $I = [0, 1]$ . Show that for any  $X$ , the set  $[X, I]$  has a single element.
  - (b) Show that if  $Y$  is path connected, the set  $[I, Y]$  has a single element.