Quiz 1, Calc 2

January 27 2006

1. If $f(x) = \dfrac{3x-5}{4x+7}$, find $f^{-1}(x)$ and state its domain and range.

Solution: To find $f^{-1}(x)$, write $y = f(x)$, switch x and y in the equation and solve for y.

$$\begin{aligned} y &= \frac{3x-5}{4x+7} \\ x &= \frac{3y-5}{4y+7} \\ x(4y+7) &= 3y-5 \\ 4xy-7x &= 3y-5 \\ 4xy-3y &= -5-7x \\ (4x-3)y &= -5-7x \\ y &= \frac{-5-7x}{4x-3} \end{aligned}$$

So $f^{-1}(x) = \dfrac{-5-7x}{4x-3}$. To find the domain of f^{-1}, we simply look at the formula we just found to see

$$\text{Domain of } f^{-1} = \text{All real numbers except } \frac{3}{4}.$$

To find the range of f^{-1}, we remember that the range of f^{-1} is equal to the domain of f. Using the formula for f, we then determine

$$\text{Range of } f^{-1} = \text{All real numbers except } -\frac{7}{4}.$$

2. Evaluate

$$\int \frac{5\sin 3x}{2-\cos 3x}dx$$

Solution: In order to evaluate this integral, we need to make a substitution. Let $u = 2-\cos 3x$. Then $du = 3\sin 3x dx$. So we have

$$\begin{aligned} \int \frac{5\sin 3x}{2-\cos 3x}dx &= \frac{5}{3}\int \frac{1}{u}du \\ &= \frac{5}{3}\ln|u| + C \\ &= \frac{5}{3}\ln|2-\cos 3x| + C. \end{aligned}$$

3. $y = \ln \dfrac{x^3}{\sqrt{x^4+3}}$. Find y'.

Solution: Using the laws of logatrithms, we can rewrite y as follows

$$\begin{aligned} y &= \ln \frac{x^3}{\sqrt{x^4+3}} \\ &= \ln x^3 - \ln \sqrt{x^4+3} \\ &= 3\ln x - \frac{1}{2}\ln(x^4+3) \end{aligned}$$

Now it is easy to find y'. By differentiating the last line above, we get

$$\begin{aligned} y' &= \frac{3}{x} - \frac{1}{2}\left(\frac{1}{x^4+3}4x^3\right) \\ &= \frac{3}{x} - \frac{2x^3}{x^4+3}. \end{aligned}$$