Quiz 2, Calc 2

February 3, 2006

1. Evaluate

$$\int_1^2 x 2^{x^2}\,dx$$

Solution: This integral simplifies after a u substitution. Set $u = x^2$. Then $du = 2xdx$, or $(1/2)du = xdx$. This gives

$$\begin{aligned}
\int_1^2 x2^{x^2}\,dx &= \frac{1}{2}\int_1^4 2^u\,du \\
&= \frac{1}{2}\left(\frac{1}{\ln 2}2^u\Big|_{u=1}^{u=4}\right) \\
&= \frac{1}{2\ln 2}2^4 - \frac{1}{2\ln 2}2^1 \\
&= \frac{16}{2\ln 2} - \frac{2}{2\ln 2} \\
&= \frac{7}{\ln 2}.
\end{aligned}$$

2. Solve

$$e^{4x^2-3} = 10.$$

Solution: To solve this equation, take the natural log of both sides and solve for x.

$$\begin{aligned}
e^{4x^2-5} &= 10 \\
\ln\left(e^{4x^2-5}\right) &= \ln 10 \\
4x^2 - 5 &= \ln 10 \\
4x^2 &= \ln 10 + 5 \\
x^2 &= \frac{\ln 10 + 5}{4} \\
x &= \pm\sqrt{\frac{\ln 10 + 5}{4}}.
\end{aligned}$$

3. Solve

$$\ln(x^3 - 8) = \ln(19).$$

Solution: Since the natural logarithm function is one-to-one (or by applying the exponential function to both sides), we get

$$\begin{aligned} x^3 - 8 &= 19 \\ x^3 &= 27 \\ x &= 3. \end{aligned}$$

4. Find y' if $y = \log_4(x^2 - 4)$. *Solution*: This is a chain rule situation. One easy

way to find the derivative of the outer function $\log_4()$ is to rewrite the function in term of the natural logarithm.

$$\log_4(x^2 - 4) = \frac{\ln(x^2 - 4)}{\ln 4}$$

Using this we find that

$$\begin{aligned} \frac{d}{dx}(\log_4(x^2 - 4)) &= \frac{d}{dx}\left(\frac{\ln(x^2 - 4)}{\ln 4}\right) \\ &= \frac{1}{(x^2 - 4)\ln 4} 2x \\ &= \frac{2x}{(x^2 - 4)\ln 4}. \end{aligned}$$