

Quiz 3, Calc 2

February 10, 2006

1. Evaluate

$$\int \frac{8x + 7}{4x^2 + 1} dx$$

Solution: In order to solve this integral, we need to break it up into two integrals using the laws of integration.

$$\int \frac{8x + 7}{4x^2 + 1} dx = \int \frac{8x}{4x^2 + 1} dx + \int \frac{7}{4x^2 + 1} dx$$

We can solve both of the integrals on the right hand side by doing a substitution. For the first integral, set $u = 4x^2 + 1$, so $du = 8xdx$. For the second integral, set $v = 2x$, so $dv = 2dx$ or $(1/2)dv = dx$. Using these substitutions, we get

$$\begin{aligned} \int \frac{8x}{4x^2 + 1} dx + \int \frac{7}{4x^2 + 1} dx &= \int \frac{1}{u} du + 7 \int \frac{1/2}{v^2 + 1} dv \\ &= \ln|u| + \frac{7}{2} \int \frac{1}{v^2 + 1} dv \\ &= \ln|u| + \frac{7}{2} \tan^{-1}(v) + C \\ &= \ln|4x^2 + 1| + \frac{7}{2} \arctan(2x) + C \end{aligned}$$

where the last line comes from undoing the substitutions.

2. Find the derivative y' if

$$y = \arcsin(\sqrt{4x})$$

Solution: This is a chain rule situation. So we have

$$\begin{aligned} y' &= \frac{d}{dx}(\arcsin(\sqrt{4x})) \\ &= \frac{1}{\sqrt{1 - (\sqrt{4x})^2}} \frac{d}{dx}(\sqrt{4x}) \\ &= \frac{1}{\sqrt{1 - 4x}} \frac{1}{2} (4x)^{-1/2} 4 \\ &= \frac{2}{\sqrt{4x}\sqrt{1 - 4x}} \\ &= \frac{1}{\sqrt{x - 4x^2}} \end{aligned}$$

The last line comes from the simplification $\sqrt{4x} = \sqrt{4}\sqrt{x} = 2\sqrt{x}$.

3. Evaluate

$$\int x^2 e^{-x} dx$$

Solution: We are integrating something which has two distinct kinds of functions involved, so we can try to solve this using integration by parts. Since polynomials simplify after differentiation, we choose $u = x^2$ and $dv = e^{-x} dx$. Therefore

$$\int x^2 e^{-x} dx = \int u dv.$$

These choices give us $du = 2x dx$ and $v = -e^{-x}$. The integration by parts formula is

$$\int u dv = uv - \int v du,$$

so we have

$$\begin{aligned} \int x^2 e^{-x} dx &= -x^2 e^{-x} - \int -e^{-x} 2x dx \\ &= -x^2 e^{-x} + 2 \int x e^{-x} dx. \end{aligned}$$

Now we need to evaluate the integral on the right hand side. In order to do that, we need to do integration by parts again. Since we are doing integration by parts twice, we need to be consistent with our choice of u and dv . So for this integral we choose $U = x$ and $dV = e^{-x} dx$. So $dU = dx$ and $V = -e^{-x}$. Therefore, we have

$$\begin{aligned} \int x^2 e^{-x} dx &= -x^2 e^{-x} + 2 \int x e^{-x} dx \\ &= -x^2 e^{-x} + 2 \left(-x e^{-x} - \int -e^{-x} dx \right) \\ &= -x^2 e^{-x} + 2 \left(-x e^{-x} + \int e^{-x} dx \right) \\ &= -x^2 e^{-x} + 2 \left(-x e^{-x} - e^{-x} \right) + C \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C. \end{aligned}$$