Quiz 5, **Calc 2**
February 24, 2006

1. Evaluate
$$\int \frac{2x^2 - 12x + 4}{x^3 - 5x^2 + 4x} dx$$

Solution: The first thing we notice is that the integrand is a proper rational function. So we want to rewrite it using partial fraction decomposition. To do this, we first need to factor the denominator of the integrand as much as possible. $x^3 - 5x^2 + 4x = x(x^2 - 5x + 4) = x(x-4)(x-1)$. This gives
$$\frac{2x^2 - 12x + 4}{x^3 - 5x^2 + 4x} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x-1}.$$
Clearing out the denominators gives the equation
$$2x^2 - 12x + 4 = A(x-1)(x-4) + Bx(x-1) + Cx(x-4).$$
Now we just plug in numbers for x to solve for A, B and C. If we plug in $x = 0$, we get $4 = 4A$ or $A = 1$. If we plug in $x = 1$, we get $-6 = -3C$ or $C = 2$. If we plug in $x = 4$, we get $-12 = 12B$ or $B = -1$. Putting this all together we get
$$\begin{aligned} \int \frac{2x^2 - 12x + 4}{x^3 - 5x^2 + 4x} dx &= \int \left(\frac{1}{x} + \frac{-1}{x-4} + \frac{2}{x-1}\right) dx \\ &= \int \frac{1}{x} dx - \int \frac{1}{x-4} dx + 2\int \frac{1}{x-1} dx \\ &= \ln|x| - \ln|x-4| + 2\ln|x-1| + C. \end{aligned}$$

Remember that the C in the last line is the constant of integration, and it doesn't have anything to do with the partial fraction decomposition.

2. Evaluate
$$\int \sec^2(\sqrt{x}) dx$$

Solution: In order to do this problem, we need to try to rewrite it. So we try a rationalizing substitution to get rid of the $\sqrt{x}$ and see if that will turn it into something we can solve. So substitute $w = \sqrt{x}$. This means $dw = \frac{1}{2\sqrt{x}} dx$ or $2\sqrt{x} dw = dx$. But we can't have two different letters on the left side, so we use the original substitution to rewrite it as $2w dw = dx$. This gives
$$\int \sec^2(\sqrt{x}) dx = \int \sec^2(w) 2w dw = 2\int w \sec^2(x) dw.$$

Now we need to solve $\int w \sec^2(w)dw$. This integral has two different types of functions in the integrand, a polynomial (w) and a trig function ($\sec^2(w)$). This means we can try integration by parts. We choose $u = w$ and $dv = \sec^2(w)dw$. Therefore $du = dw$ and $v = tan(w)$. So integration by parts gives

$$\begin{aligned} \int w \sec^2(x)dw &= w\tan(w) - \int \tan(w)dw \\ &= w\tan(w) - \ln|\sec(w)| + C \end{aligned}$$

By putting everything together, we get

$$\begin{aligned} \int \sec^2(\sqrt{x})dx &= 2\int w\sec^2(x)dw \\ &= 2\left(w\tan(w) - \ln|\sec(w)|\right) + C \\ &= 2\sqrt{x}\tan(\sqrt{x}) - 2\ln|\sec(\sqrt{x})| + C. \end{aligned}$$