

Quiz 7, Calc 2

March 31, 2006

1. Evaluate

$$\lim_{n \rightarrow \infty} \frac{4n + 6}{\sqrt{9n^2 + 7}}$$

Solution: To evaluate this limit, we can divide the top and bottom by n .

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{4n + 6}{\sqrt{9n^2 + 7}} &= \lim_{n \rightarrow \infty} \frac{\frac{4n+6}{n}}{\frac{\sqrt{9n^2+7}}{n}} \\ &= \lim_{n \rightarrow \infty} \frac{4 + \frac{6}{n}}{\sqrt{9 + \frac{7}{n^2}}} \\ &= \frac{4 + 0}{\sqrt{9 + 0}} \\ &= \frac{4}{3}. \end{aligned}$$

2. Evaluate

$$\lim_{n \rightarrow \infty} \frac{\sin(n)}{n + 1}$$

Solution1: The first thing to notice here is that $\lim_{n \rightarrow \infty} \sin(n)$ doesn't exist since $\sin(n)$ is oscillating between 1 and -1. However, we can use this fact to set up $-1 \leq \sin(n) \leq 1$. This gives

$$\frac{-1}{n + 1} \leq \frac{\sin(n)}{n + 1} \leq \frac{1}{n + 1}.$$

Since

$$\lim_{n \rightarrow \infty} \frac{-1}{n + 1} = \lim_{n \rightarrow \infty} \frac{1}{n + 1} = 0$$

we can use the Squeeze Theorem to show

$$\lim_{n \rightarrow \infty} \frac{\sin(n)}{n + 1} = 0.$$

3. Decide if the following series are convergent or divergent. (Bonus: If the series converges, what does it converge to?)

(a) $\sum_{n=1}^{\infty} \arctan(n)$

$$(b) \sum_{n=2}^{\infty} 4^{2-n} 3^{n-1}$$

Solution (a): For this series (and for any series), the first test to try is the test for divergence. So consider

$$\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2} \neq 0.$$

So this series diverges by the test for divergence.

Solution (b): For this series, notice that the only place n appears in the term is in the exponent. This makes me think of geometric series. So we can write out the first few terms of the series to see if it is geometric.

$$\sum_{n=2}^{\infty} 4^{2-n} 3^{n-1} = 3 + \frac{9}{4} + \frac{27}{16} + \frac{81}{64} + \dots$$

From this we see that the series is geometric with common ratio $3/4$. So since $3/4 < 1$, the series converges and in fact it converges to

$$\sum_{n=2}^{\infty} 4^{2-n} 3^{n-1} = \frac{3}{1 - \frac{3}{4}} = 12.$$