Simon Joyce MT222-Section 04: Assignment 7

Solutions to be submitted in lecture Friday 04/03/09. Your solutions to each question should be neatly written up on a separate piece of paper and your solutions stapled or clipped together.

Find the radius and interval of convergence of the following power series.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n(n+1)}$
   answer: $R = 1$ *and* $-1 \leq x \leq 1$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-4)^n}{n^2}$
   answer: $R = 1$ *and* $3 \leq x \leq 5$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$
   answer: $R = \infty$

4. Given $f(x) = \dfrac{4}{4+x^2}$. Find a power series representation for $f(x)$ and determine the interval of convergence.
   answer: $f(x) = \displaystyle\sum_{n=0}^{\infty} \left[-\left(\frac{x}{2}\right)^2\right]^n$ *and* $-2 < x < 2$

5. Given $f(x) = \dfrac{x^2}{(1-x)^2}$. Find a power series representation for $f(x)$ and determine the radius of convergence.
   answer: $f(x) = \displaystyle\sum_{n=2}^{\infty} (n-1)x^n$ *and* $R = 1$.