MATH 371 - Summer 2007 Final Exam Name:_____

Show all work for full credit. You may leave your answers in implicit form.
Warning: If, for a problem, a method is specified and you do not use it, you will receive no credit.

1. (6,6) Solve the separable differential equations:
   (a) $\frac{dy}{dx} = e^{3x+2y}$
   (b) $3y^2 \cos(x) y' = \sin(x)$

2. (8) Solve the equation $y' = \sec(\frac{y}{x}) + \frac{y}{x}$ using the substitution $y = ux$.

3. (8) Find the general solution to the system of equations: $\begin{aligned} y_1' &= 2y_1 - 3y_2 \\ y_2' &= y_1 - 2y_2 \end{aligned}$

4. (5,8,2) Given the equation $(4y^2 + x)dx + 4xydy = 0$,
   (a) Show that this equation is *not* exact.
   (b) Find an integrating factor to make it exact.
   (c) Show that the new equation *is* exact. **Do Not Solve.**

5. (8,2) Solve the differential equation $y'' + 6y' + 9y = t^2e^{-3t}$, with initial values $y(0) = 0$ and $y'(0) = 0$, using Laplace transforms. Verify that your answer y satisfies the equation and has the specified initial values.

6. (6,6,6,2) Given the differential equation $y'' + 2y' - 15y = 4e^{5x}$,

   (a) Find y_h.

   (b) Find y_p using variation of parameters.

   (c) Find y_p using undetermined coefficients.

   (d) Give the general solution.

7. (6,9) Given the equation $y' + 5y = e^{-5x}$ with initial value $y(0) = 1$,

   (a) Solve the initial value problem by finding an integrating factor.

   (b) Solve using Laplace transforms.

8. (5,2,5) Euler-Cauchy equations.

   (a) Solve the equation $x^2y'' + 6xy' - 6y = 0$.

   (b) Verify your answer in part a.

   (c) Solve the equation $x^2y'' - 17xy' + 81 = 0$.