

M371 - Quiz 5

Name: KeyUse substitution to solve $\frac{dy}{dx} = \frac{x+3y}{3x+y}$.Let $y = ux$, so $y' = u'x + u$.

$$u'x + u = \frac{x+3(ux)}{3x+(ux)} = \frac{1+3u}{3+u}$$

$$u'x = \frac{1+3u}{3+u} - u = \frac{1+3u}{3+u} - \frac{u^2+3u}{3+u}$$

$$u'x = \frac{3u+1-u^2-3u}{3+u} = \frac{1-u^2}{3+u}$$

$$\int \frac{3+u}{1-u^2} du = \int \frac{dx}{x} = \ln|x| + c_1$$

$$\int \frac{2}{1-u} du + \int \frac{1}{1+u} du = \ln|x| + c_1 \quad (\text{partial fractions})$$

$$-2 \ln|1-u| + \ln|1+u| = \ln|x| + c_1$$

$$\ln \left| \frac{u+1}{(u-1)^2} \right| = \ln|x| + c_1$$

$$\frac{u+1}{(u-1)^2} = cx$$

$$\frac{u+1}{x(u-1)^2} = c$$

$$\frac{(y/x)+1}{x((y/x)-1)^2} = c$$

$$\frac{y+x}{x^2((y/x)-1)^2} = c$$

$$\frac{y+x}{(y-x)^2} = c$$