Review Problems of Math 222, Name_____,

Section_____, Instructor_____, SSN_____

A. (Maybe 30% of the total points).

1. Complete the following items (10 points):

1. The form of the partial fraction of $\frac{1}{x^3(x^2+x+1)^2(x-2)}$ (**no need to solve the coefficients**) is _____ (**key:** $\frac{a_1}{x}+\frac{a_2}{x^2}+\frac{a_3}{x^3}+\frac{b_1+c_1x}{x^2+x+1}+\frac{b_2+c_2x}{(x^2+x+1)^2}+\frac{A}{x-2}$)
2. $\int udv = uv$ _____ (**key:** $-\int vdu$)
3. $\frac{d}{dx}f(u(x)) = \frac{d}{du}$_____$\frac{d}{dx}$ _____ (**key:** f, u)
4. If f is continuous, then $\frac{d}{dx}\int_o^x f(t)dt =$ _____ (**key:** $f(x)$)
5. $g(g^{-1}(x)) =$ _____ and $(g^{-1}(x))' =$ _____ (**key:** x, $\frac{1}{g'(g^{-1}(x))}$)
6. $\frac{d}{dx}\cos^{-1}x =$ _____ (**key:** $\frac{-1}{\sqrt{1-x^2}}$)
7. $\frac{d}{dx}\cot^{-1}x =$ _____ (**key:** $\frac{-1}{1+x^2}$)
8. $\frac{d}{dx}\sin^{-1}(g(x)) =$ _____ (**key:** $\frac{g'(x)}{\sqrt{1-(g(x))^2}}$)
9. $\frac{d}{dx}\sec^{-1}x =$ _____ (**key:** $\frac{1}{x\sqrt{x^2-1}}$)
10. $\frac{d}{dx}\sec x =$ _____ (**key:** $\sec x\tan x$)
11. $\int a^x dx =$ _____ (**key:** $\frac{a^x}{\ln a}+c$)
12. $\int \sin x dx =$ _____ (**key:** $-\cos x + c$)
13. $\int \sec x dx =$ _____ (**key:** $\ln|\sec x+\tan x|+c$)
14. $\int \csc x dx =$ _____ (**key:** $\ln|\csc x-\cot x|+c$)
15. $\int \tan x dx =$ _____ (**key:** $\ln|\sec x|+c$)
16. $\int \sec^2 x dx =$ _____ (**key:** $\tan x + c$)
17. $\int x^{-1}dx =$ _____ (**key:** $\ln|x|+c$)
18. If $n\neq -1$, then $\int x^n dx =$ _____ (**key:** $\frac{x^{n+1}}{n+1}+c$)
19. $\sinh(x) =$ _____. (**key:** $\frac{e^x-e^{-x}}{2}$)
20. $\frac{d}{dx}\tanh x =$ _____ (**key:** $sech^2x$)
21. $\frac{d}{dx}\sinh x =$ _____ (**key:** $\cosh x$)
22. $\int_1^\infty \frac{1}{x^p}dx$ is convergent if and only if p_____ (**key:** >1)
24. Under proper assumptions, $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}$ _____ if the limit is of $\frac{0}{0}$ type. (**key:** $\frac{f'(x)}{g'(x)}$)
25. Strategy for evaluating $\int \sin^m x\cos^n x dx$, if n is odd, then use substitution $u =$ _____ and identity $\cos^2 x =$ _____ (**key:** $\sin x$, $1-\sin^2 x$)
26. Strategy for evaluating $\int \sin^m x\cos^n x dx$, if m is odd, then use substitution $u =$ _____ and identity $\sin^2 x =$ _____ (**key:** $\cos x$, $1-\cos^2 x$)
27. Strategy for evaluating $\int \sin^m x\cos^n x dx$, if n and m are even, then use identities $\begin{cases}\cos^2 x = \frac{1}{2}(\ \_\_\_\_\_\\ \sin^2 x = \frac{1}{2}(\ \_\_\_\_\_\end{cases}$ (**key:** $\begin{cases}1+\cos 2x)\\ 1-\cos 2x)\end{cases}$)
28. Strategy for evaluating $\int \tan^m x\sec^n x dx$, if m is odd, use substitution $u =$ _____ (**key:** $\sec x$)
29. Strategy for evaluating $\int \tan^m x\sec^n x dx$, if n is even, use substitution $u =$ _____ (**key:** $\tan x$)

30. Table of Trigonometric Substitution in integration. Do not write down the domain.

expression	*substitution*	*identity*
$\sqrt{a^2-x^2}$	$x=$	
$\sqrt{a^2+x^2}$	$x=$	
$\sqrt{x^2-a^2}$	$x=$	

(**key:** $\begin{cases} a\sin\theta, & \sin^2\theta+\cos^2\theta=1 \\ a\tan\theta, & \tan^2\theta+1=\sec^2\theta \\ a\sec\theta, & \tan^2\theta+1=\sec^2\theta \end{cases}$)

31. The arc length function of curve $y=f(x)$ is $s(t)=\int_a^t$ _____
(**key:** $\sqrt{1+(f'(x))^2}dx$)

32. Area of surface of revolution if rotating about $x-$axis $S=\int_a^b$ _____
Area of surface of revolution if rotating about $y-$axis $S=\int_c^d$ _____
(**key:** $2\pi y\sqrt{1+(\frac{dy}{dx})^2}dx$, $2\pi x\sqrt{1+(\frac{dx}{dy})^2}dy$)

The following questions are related to the parametric equations of the curve C: $x=f(t)$ and $y=g(t)$, $\alpha\le t\le\beta$:

33. $\frac{dy}{dx}=$ _____ and $\frac{d^2y}{dx^2}=$ _____. (**key:** $\frac{g'(t)}{f'(t)}$, $\frac{\frac{d}{dt}\frac{dy}{dx}}{f'(t)}$)

34. The area between the curve C and the x-axis is $A=\int_\alpha^\beta$ _____dt
(**key:** $g(t)f'(t)$)

35. The arc length of the curve C is $L=\int_\alpha^\beta$ _____dt
(**key:** $\sqrt{(f'(t))^2+(g'(t))^2}$)

36. The surface area is $S=\int_\alpha^\beta$ _____dt if C rotates about x-axis.
(**key:** $2\pi g(t)\sqrt{(f'(t))^2+(g'(t))^2}$)

37. The surface area is $S=\int_\alpha^\beta$ _____dt if C rotates about y-axis.
(**key:** $2\pi f(t)\sqrt{(f'(t))^2+(g'(t))^2}$)

The following questions are related to the Polar coordinates and a polar curve C: $r=f(\theta)$, $a\le\theta\le b$.

38. $\frac{dy}{dx}=$ _____ (**in terms of r and θ only**).
(**key:** $\frac{r'(\theta)\sin\theta+r(\theta)\cos\theta}{r'(\theta)\cos\theta-r(\theta)\sin\theta}$)

39. The area of the polar region bounded by the curve C and the rays $\theta=a$ and $\theta=b$ is $A=\int_a^b$ _____$d\theta$. (**key:** $\int_a^b\frac{1}{2}r^2d\theta$)

40. The arc length of the curve C is $L=\int_a^b$ _____$d\theta$ (**key:** $\int_a^b\sqrt{r^2+(\frac{dr}{d\theta})^2}d\theta$)