

Solve $ydx = 2(x + y)dy$ using the substitution $y = ux$.

$$\begin{aligned} uxdx &= 2(x + ux)(xdu + udx) \\ udx &= 2(u + 1)xdu + 2(u + 1)udx \\ (u - 2u^2 - 2u)dx &= (2u + 2)xdu \\ \frac{-2u^2 - u}{2u + 2}dx &= xdu \\ \frac{dx}{x} &= -2\left(\frac{u + 1}{2u^2 + u}\right)du \end{aligned}$$

$$\text{PartialFractions : } \frac{1}{u} + \frac{-1}{2u + 1} = \frac{u + 1}{2u^2 + u}$$

$$\begin{aligned} -\ln|x| + c_1 &= 2\left(\frac{1}{u} + \frac{-1}{2u + 1}\right)du \\ -\ln|x| + c_1 &= 2(\ln|u| - \frac{1}{2}\ln|2u + 1|) \\ -\ln|x| + c_1 &= \ln\left|\frac{u^2}{2u + 1}\right| \\ c_1 &= \ln\left|\frac{xu^2}{2u + 1}\right| \\ c_1 &= \ln\left|\frac{(y^2/x)}{2(y/x) + 1}\right| \\ c &= \frac{x^{-1}y^2}{2x^{-1}y + 1} \\ c &= \frac{y^2}{2y + x} \end{aligned}$$