Math 222 Fall 2010 Test II Name________________

The exam has 166 points - show all your work. You may use backs of pages if necessary.

No calculators nor electronic devices.

[8] 1. State the definition: $\lim_{n\to\infty} a_n = L$ where $\{a_n\}_{n=1}^{\infty}$ is a sequence of real numbers and L is a real number.

[30] 2. Evaluate the following - no justification required.

(i) $\lim_{n\to\infty} \frac{3^{n+2}}{4^n} =$

(ii) $\lim_{n\to\infty} 3(n)^{2/n} =$

(iii) $\lim_{n\to\infty} (1 - 2/n)^n =$

(iv) $\sum_{n=1}^{\infty} \frac{2^n \pi}{3^{n+1}}$

(v) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

(vi) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n+1}(2n+1)!}$

[16] 3. Calculate the following derivatives - you need not simplify

(i) $f(x) = \sum_{n=1}^{\infty} (\tan n)(\sin x)^{2n}$

$f'(x) =$

(ii) $g(x) = (\sin^{-1} x)\left(\sum_{n=0}^{\infty} \frac{n x^{n+1}}{n^2+1}\right)$

$g'(x) =$

[16] 4. Evaluate the integrals (You may leave answers in series notation).

(i) $\int_0^1 \cos x^2 dx$

(ii) $\int \sinh\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}\right) \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}\right) dx$

[10] 5. State in <u>full</u> the Integral Test for convergence of the series $\sum_{n=1}^{\infty} a_n$.

[30] 6. Determine whether the following series is convergent. Sketch the reasoning for your conclusions.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^n + 5^n}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+(1/n)}}$

(iii) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^8 + 1}}$

[20] 7. Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n (x-1)^{2n+1}}{n}$.

[21] 8. For each of the following, find Taylor series about 0 for the following - give the radius of convergence.

(i) $\frac{x}{3+x^3}$

(ii) $(x^2+1)e^{x^2}$

(iii) $\int_0^x e^{t^2}\,dt$

[8] 9. Give an error bound on estimating $1/e$ by $\sum_{n=0}^{5} \frac{(-1)^n}{n!}$. Choose any estimate of remainder, but justify its use.

[8] 10. Suppose f is a function, $f(1) = -1$, $f(2) = 2$ and $\lim_{x \to \infty} f(x) = 1$, evaluate $\sum_{n=1}^{\infty} 2f(n) - f(n+1) - f(n+2)$.