Determine whether the series converges or diverges. Show all your work and identify any tests or series you are using, and also show the hypothesis of the test are satisfied.

1. $\sum_{n=5}^{\infty} \frac{\pi^n}{n^2}$

$$\lim_{n\to\infty} \frac{\pi^n}{n^2} = \lim_{x\to\infty} \frac{\pi^x}{x^2} \overset{H}{=} \lim_{x\to\infty} \frac{\pi^x \ln x}{2x}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\pi^x (\ln\pi)^2}{2} = \infty \neq 0.$$

Diverges by the test for divergence

Many methods work - Limit comp, integral, etc

2. $\sum_{n=1}^{\infty} \frac{e}{n^2+4}$ $\quad a_n = \frac{e}{n^2+4} > 0 \quad b_n = \frac{e}{n^2} > 0$

$$n^2+4 \geq n^2$$

Thus $\frac{e}{n^2+4} \leq \frac{e}{n^2}$

$$\sum_{n=1}^{\infty} \frac{e}{n^2} = e\sum_{n=1}^{\infty} \frac{1}{n^2} \quad \text{converges}$$

since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series. Hence $\sum_{n=1}^{\infty} \frac{e}{n^2+4}$ converges by comparison

3. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$

$$\underbrace{\frac{2+(-1)^n}{n\sqrt{n}}}_{=a_n>0} \leq \underbrace{\frac{3}{n^{3/2}}}_{=b_n>0}$$

$\sum \frac{3}{n^{3/2}}$ converges because

$\sum \frac{1}{n^{3/2}}$ is a convergent p-series.

Thus $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$ converges by comparison.

4. $\frac{1}{4} - \frac{1}{5} + \frac{4}{25} - \frac{16}{125} + \frac{64}{625} - \frac{256}{3125} + \cdots$

$$= \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{-4}{5}\right)^{n-1}$$

Converges because it is a convergent geometric series $\left(|r| = \frac{4}{5} < 1\right)$.

1. Determine if the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(-1)^{n-1} n}{n^2+1} = \frac{(-1)^{n-1} \frac{1}{n}}{1+\frac{1}{n^2}}$$

$-1 \leq (-1)^{n-1} \leq 1$ for all $n \in \mathbb{N}$

$\frac{-1}{n} \leq \frac{(-1)^{n-1}}{n} \leq \frac{1}{n}$ for all $n \in \mathbb{N}$ since $n > 0$

$$\lim_{n\to\infty} \frac{-1}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

Thus by the Squeeze thm

$$\lim_{n\to\infty} \frac{(-1)^{n-1}}{n} = 0$$

Thus $\lim_{n\to\infty} a_n = \frac{0}{1} = 0.$

2. Find the area of the surface obtained by rotating the curve about the x-axis.

$y = \sqrt{1+4x}$ $\quad 1 \leq x \leq 5$

$$SA = \int 2\pi y \, ds = \int 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_1^5 2\pi \sqrt{1+4x} \sqrt{1 + \left(\frac{4}{2\sqrt{1+4x}}\right)^2} \, dx$$

$$= \int_1^5 2\pi \sqrt{1+4x} \sqrt{1 + \frac{4}{1+4x}} \, dx$$

$$= \int_1^5 2\pi \sqrt{(1+4x)\left(1 + \frac{4}{1+4x}\right)} \, dx$$

$$= \int_1^5 2\pi \sqrt{1+4x+4} \, dx$$

$$= \int_1^5 2\pi \sqrt{5+4x} \, dx \qquad u = 5+4x$$

$$du = 4\,dx$$

$$= \frac{1}{2}\pi \int_9^{25} \sqrt{u} \, du$$

$$= \frac{1}{2}\pi \left[u^{3/2}\right]_9^{25} = \frac{1}{2}\pi \left[25^{3/2} - 9^{3/2}\right]$$

3. (a) What is a sequence?

An ordered list of numbers OR

A function with domain $\mathbb{N}$.

(b) What is a series?

A series is the sum of a sequence of numbers

various answers ok

4. Give an example of a sequence that is

(i) bounded $\{(-1)^n\}_{n=1}^{\infty}$

(ii) increasing $\{n\}_{n=1}^{\infty}$

(iii) convergent $a_n = \frac{(-1)^n n}{n^2+1}$