Research Interests

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My research experience lies primarily in the area of finite group theory. My dissertation research is a study of finite complete groups, focusing on finite complete wreath products.

For an element x in a group G, conjugation by x is an automorphism (denoted τ_x) of G. Such automorphisms are known as *inner automorphisms*, and the set of inner automorphisms is denoted by Inn G. If Aut G denotes the automorphism group of G then

$$\text{Inn } G = \{\tau_x \in \text{Aut } G \mid x \in G;\ \text{for all } g \in G,\ \tau_x(g) = x^{-1}gx\}.$$

It turns out that Inn G is more than just a set; Inn G is a normal subgroup of Aut G.

A *complete group* is a group with a trivial center for which Inn G equals Aut G. The symmetric group acting on n points, where n is a positive integer at least 3 and not equal to 6, is an example of a complete group. While the proof for larger n relies on a result of simple groups, the proof of this fact for $n = 3$ is a straightforward calculation.

It's easy to pick out a homomorphism from G into Inn G, defined by mapping x to τ_x. The kernel of this homomorphism is the center of G, so a centerless group is isomorphic to its group of inner automorphisms. Thus a complete group G is isomorphic to Aut G. In this sense complete groups have a "minimal" automorphism group – every element of the group determines a unique automorphism and there are no other automorphisms.

One reason complete groups are so interesting is that every finite group can be embedded in a finite complete group [2]. Moreover, every finite centerless group can be embedded subnormally in a complete group [3].

Every finite group can also be embedded in a finite wreath product [1]. A finite wreath product is a semidirect product $N \rtimes H$ wherein the normal subgroup N is a direct product of n copies of a finite group G and the finite complement H acts with degree n by permuting the components of N.

I've recently determined when a finite wreath product is complete, using cohomology and permutation group theory. In addition I've classified complete Frobenius groups, another type of semidirect product, and have worked on a new proof for the determination of when the base of a wreath product is characteristic.

In the future I intend to continue my study of complete groups and automorphisms. As part of my master's thesis, I studied character theory and some graph theory. I find both areas interesting and would like to find useful application of their techniques to the study of automorphisms.

I have also developed an interest in expanding my results to the area of infinite group theory. There are several families of infinite groups that I suspect harbor complete groups. Candace Schenk, another Binghamton University graduate student, and I have been studying the current research of Thompson's groups and the Baumslag-Solitar groups to determine whether or not these groups could be complete.

Outside of group theory, I retain an interest in biostatistics and its applications in the area of epidemiology from when I was a student at the University of Illinois at Chicago's School of Public Health. Admittedly my experience in this area is much more limited than my work in abstract mathematics. However, while in Chicago I computed and interpreted statistics for a cross-sectional study of meningioma, a type of brain tumor. I also worked to increase public awareness of a kidney cancer study, with the goal of increasing the response rate in the control group. These two experiences have provided me with insight into how I might rejuvenate my connections in the field and pursue new research experiences to supplement my primary research in group theory.

References

[1] John D. Dixon and Brian Mortimer, *Permutation groups*, Springer, 1996.

[2] J.S. Rose, *A subnormal embedding theorem for finite groups*, J. London Math. Soc. **5** (1972), 253 – 259.

[3] Helmut Wielandt, *Eine Verallgemeinerung der invarianten Untergruppen*, Math. Z. **45** (1939), 209 – 244.