

Math 220 – Review of Exponential Functions (Answers Below)

I. On the same set of axes, plot some points to sketch the graphs of $f(x) = 2^x$ and $g(x) = 3^x$. For which values of x is $f > g$? Without plotting points, use these graphs to sketch (on the same axes) the graph of $h(x) = e^x$.

II. On the same set of axes, sketch the graphs of $f(x) = 2^x$ and $g(x) = 2^{-x}$. Compare your graphs. Can you suggest a generalization for comparing the functions $h(x) = a^x$ and $k(x) = \left(\frac{1}{a}\right)^x$ for any value $a > 1$?

III. For each of the functions f , g , h in problems I and II, what is the domain of the function? What is the range? What point do they all have in common?

IV. Solve for x .

1. $x^2e^x - 5xe^x + 4e^x = 0$

2. $x^2e^{-3x} + 7e^{-3x} = 0$

3. $5^{2x} - 5^{-x+1} = 0$

4. $9^{x-1} = 3^{1+x}$

V. Find the natural domain for each function.

1. $f(x) = e^{\frac{x-2}{x^2-3x+2}}$

2. $f(x) = e^{\sqrt{x-1}} + e^{-\sqrt{2-x}}$

3. $f(x) = \frac{7^{x^2} + 3x \cdot 6^x}{5^{x^2-1}}$

4. $f(x) = \frac{7^{x^2} + 3x \cdot 6^x}{5^{x^2-1} - 25}$

Answers:

I. $f > g$ for all x in $(-\infty, 0)$. The graph of h should be between the graphs of f and g .

II. The graphs of f and g should be mirror images of each other (symmetric) over the y -axis.

It is true for any function f that the graphs of $f(x)$ and $f(-x)$ are symmetric over the y -axis (assuming that both x and $-x$ are in the domain of f). We have this situation with h and k because $\left(\frac{1}{a}\right)^x = a^{-x}$.

III. Domain: \mathfrak{R} Range: $(0, \infty)$ Point in common: $(0, 1)$, the y -intercept

IV. 1. $x=1$ or $x=4$ 2. No solutions 3. $\frac{1}{3}$ 4. 3

V. 1. $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$ 2. $[1, 2]$ 3. \mathfrak{R} 4. $x \neq \sqrt{3}$ and $x \neq -\sqrt{3}$