Math 220 Fall 2009 Test 2 J. Brewster Name Key - p.m. class

You must show your work for all but "short answer" questions.
No calculators or other aids are permitted for this test.
There are 100 points available for this test.
Problems are printed on both sides of the test, except for the last page.

1. (12 pts.) State the <u>definition</u> of "derivative function."

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Use the definition to find $f'(x)$ for $f(x)=\sqrt{x}+5$. No credit will be given for any other method of finding the derivative.

$$\lim_{h\to 0} \frac{(\sqrt{x+h}+5)-(\sqrt{x}+5)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) = \lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

<u>NOTE</u>: For all further questions on this test, you may use the derivative rules to find any needed derivatives.

2. (6 points) Given $f(x)=x^3-5$, show that the line tangent to the graph of f at the point (2, 3) is parallel to the line tangent to the graph of f at the point (–2, –13).

$f'(x) = 3x^2$

$f(-2) = 3(-2)^2 = 12$ is the <u>slope</u> of the line tangent to f at $(-2,-13)$.

$f(2) = 3(2)^2 = 12$ is the <u>slope</u> of the line tangent to f at $(2,3)$.

Since these lines have the same slope, they are parallel.

3. (31 points). For each of the following functions, use the derivative rules to find the derivative. ***DO NOT SIMPLIFY!!!***

$f(x) = 5x^3 - \frac{1}{x^2} + \frac{x}{6} - \sqrt{x} + 8 \quad = 5x^3 - x^{-2} + \frac{1}{6}x - x^{1/2} + 8$

$f'(x) = 15x^2 + 2x^{-3} + \frac{1}{6} - \frac{1}{2}x^{-1/2}$

$f(x) = e^x + \ln x + 2^x + \log_3 x + \ln 5$

Note: $\ln 5$ is a constant, so its derivative is 0.

$f'(x) = e^x + \frac{1}{x} + 2^x \ln 2 + \frac{1}{x \ln 3}$

$f(x) = \frac{1}{\sqrt{x^5 + 2x}} \quad = (x^5 + 2x)^{-1/2}$

$f'(x) = -\frac{1}{2}(x^5 + 2x)^{-3/2}(5x^4 + 2)$

$f(x) = \frac{3x^7 + x^4}{2x^2 - x + 5}$

$f'(x) = \frac{(21x^6 + 4x^3)(2x^2 - x + 5) - (4x - 1)(3x^7 + x^4)}{(2x^2 - x + 5)^2}$

$f(x) = (3x - e^x)(x^3 - 2x + 1)$

$f'(x) = (3 - e^x)(x^3 - 2x + 1) + (3x^2 - 2)(3x - e^x)$

$f(x) = \ln\left(5x + \sqrt{e^x - 7}\right) \quad = \ln\left(5x + (e^x - 7)^{1/2}\right)$

$f'(x) = \frac{1}{5x + \sqrt{e^x - 7}} \cdot \left(5 + \frac{1}{2}(e^x - 7)^{-1/2} e^x\right)$

4. (8 points)

$$f(x)=\begin{cases} x^2+5x+3 & x\le 0 \\ 3x+4 & 0<x<6 \\ x^2-x & x\ge 6 \end{cases}$$

(a) At which values in the domain of f is f not differentiable? Justify your answer.

f is not differentiable at $x=0$ because it is not continuous at $x=0$ (LHL$=3$, RHL$=4$). For f to be differentiable at $x=0$ it must be continuous at $x=0$.

f is not differentiable at $x=6$ because the left-hand derivative is not equal to the right-hand derivative.

LHD $=3$, RHD $=11$

(b) What is $f'(x)$?

$$f'(x)=\begin{cases} 2x+5 & x<0 \\ 3 & 0<x<6 \\ 2x-1 & x>6 \end{cases}$$

5. (10 points) Given equation $x^2y^3+2x-7=\dfrac{1}{4x+y-3}$, find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(x^2y^3+2x-7\right)=\frac{d}{dx}\left(4x+y-3\right)^{-1}$$

$$2xy^3+3y^2\frac{dy}{dx}\cdot x^2+2=-(4x+y-3)^{-2}\left(4+\frac{dy}{dx}\right)$$

$$2xy^3+3y^2\frac{dy}{dx}x^2+2=-4(4x+y-3)^{-2}-(4x+y-3)^{-2}\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx}x^2+(4x+y-3)^{-2}\frac{dy}{dx}=-4(4x+y-3)^{-2}-2xy^3-2$$

$$\frac{dy}{dx}\left(3y^2x^2+(4x+y-3)^{-2}\right)=-4(4x+y-3)^{-2}-2xy^3-2$$

$$\frac{dy}{dx}=\frac{-4(4x+y-3)^{-2}-2xy^3-2}{3y^2x^2+(4x+y-3)^{-2}}$$

6. (11 points) Jack has hired Jill to get rid of the ants in his apartment building. Jill has devised a treatment that she claims will fix the problem within thirty days. She estimates that the number N of ant nests present at time t days from now is given by the function $N(t) = -t^2 + 20t + 290$. The answers to questions (b) and (c) below require units of measure.

(a) What is the current quantity of ant nests?

$N(0) = 290$ nests

(b) What is the average rate of change in the quantity of nests from day 5 to day 20 of treatment?

$$\frac{N(20) - N(5)}{20-5} = \frac{(-400+400+290) - (-25+100+290)}{15}$$

$$= \frac{-\cancel{400}+\cancel{400}+\cancel{290}+25-100-\cancel{290}}{15} = -\frac{75}{15} = -5 \text{ Nests/day}$$

(c) What is the rate of change of the quantity of nests on day 25?

$N'(t) = -2t+20$

$N'(25) = -50+20 = -30$ Nests/day

(d) If Jill's estimate is correct, will the ants be gone within thirty days? Support your answer.

$N(30) = -900+600+290 = -10$

Yes.

7. (5 points) Given that $\frac{d}{dx}|x| = \frac{x}{|x|}$, find $\frac{d}{dx}|x^2 - e^x|$

$$\frac{d}{dx}|x^2-e^x| = \frac{x^2-e^x}{|x^2-e^x|} \cdot (2x-e^x)$$

(Chain Rule $u = x^2-e^x$)

8. (7 points) Suppose $g(x) = xe^x$.

$g'(x) = 1 \cdot e^x + e^x x = e^x + e^x x$

$g''(x) = e^x + e^x x + e^x = 2e^x + e^x x$

$g'''(x) = 2e^x + e^x x + e^x = 3e^x + e^x x$

$g^{(73)}(x) = 73e^x + e^x x$

9. (10 points) It is estimated that when the circulation of a certain newspaper is x thousand, the annual advertising revenue received by the newspaper will be $R(x) = \frac{1}{2}x^2 + 3x + 160$ thousand dollars. The circulation of the paper is currently 10,000 ($x = 10$) and is increasing at a rate of 2,000 per year. At what rate will the advertising revenue be changing, with respect to time, three years from now? Include proper units of measure in your answer.

Want: $\frac{dR}{dt}$ when $t = 3$ yrs.

Know: $\frac{dx}{dt} = 2$ thousand papers/yr.

When $t = 3$, $x = 10 + 2(3) = 16$ thousand papers.

$R = \frac{1}{2}x^2 + 3x + 160$

$\frac{d}{dt}R = \frac{d}{dt}\left(\frac{1}{2}x^2 + 3x + 160\right)$

$\frac{dR}{dt} = x\frac{dx}{dt} + 3\frac{dx}{dt}$

$\frac{dR}{dt} = 16(2) + 3(2) = 32 + 6 = 38$ thousand dollars/yr.