Math 220 -- Logarithm Review and Practice Sheet (Answers Below)

ALL OF THESE EXERCISES ARE TO BE DONE WITHOUT A CALCULATOR.

I. Find the exact value of each expression:

1. $\log_2 64$ 2. $\log_6 \frac{1}{36}$ 3. $\log_8 2$ 4. $e^{3\ln 2}$

5. $\ln e^{\sqrt{2}}$ 6. $\log_5 10 + \log_5 20 - 3\log_5 2$ 7. $2^{(\log_2 3 + \log_2 5)}$

II. Find x:

1. $\log_x e^5 = 5$ 2. $\log_{\frac{1}{4}} 16 = x$ 3. $\log_x 9 = -2$

4. $\log_{16} x = \frac{3}{2}$ 5. $10^{\log_{10} x} = 33$ 6. $x = \log_4 2 + \log_2 4$

III. Solve for x. Do not use a calculator (an answer like $\frac{\ln 2}{7}$ is OK).

1. $25 = 5(2^x)$ 2. $\ln x = 0$ 3. $\ln e^{x^2} = 4$ 4. $.01 = e^{-.05x}$

5. $e^{\ln 4x} = 16$ 6. $\ln e^{3x} = 12$ 7. $2 = (1+x)^{12}$ 8. $\ln x = -1$

9. $\ln(2x-1) = 3$ 10. $e^{3x-4} = 2$ 11. $2^{x-5} = 3$ 12. $\ln x + \ln(4x) = 2$

13. $\ln(3x^2) - \ln(9x) = 1$ 14. $\ln(x-9) + \ln x = 3$ 15. $\log_{10} 5 + \log_{10} x = 2$

16. $\log_{10} x - \log_{10} 8 = 1$ 17. $\log_4 x = \log_2 5$ 18. $\frac{\ln 1}{\ln 5} = x + 3$

19. $\ln(\ln x) = 0$ 20. $\ln(\ln x) = 1$ 21. $\log_4(x+6) = -\frac{1}{2}$

22. $\log(x+6) - \log(x-3) = 1$ 23. $2\ln(x-1) = \ln(x^2-5)$ 24. $\log_2(x+2) + \log_2(x-1) = 2$

25. $7^{2x} - 4 = 0$ 26. $e^{2x} + e^x - 6 = 0$ 27. $\ln(a+b) + \ln(a-b) - \ln(a^2-b^2) = x$

IV. Show that $(\log_a b)(\log_b a) = 1$ for $a,\ b > 0;\ a,\ b \neq 1$

V. Express as a single logarithm:

1. $\ln 5 + 2\ln 3 - \frac{1}{2}\ln 4$ 2. $\log_4(x) + \frac{1}{5}\log_4(x-y) - 2\log_4(z)$

VI. Use the rules for logarithms to expand the quantity:

1. $\ln\left(\frac{10c^8}{\sqrt[4]{ab^3}}\right)$ 2. $\log_2\sqrt{a\left(b^2+c^2\right)}$

VII. Find the natural domain for each function:

1. $f(x)=\ln\sqrt{7+x}$ 2. $f(x)=\ln(x-3)-\log_2(10-x)$ 3. $f(x)=\frac{6}{\ln(x+5)}$

4. $f(x)=\ln(x-3)+\ln(2-x)$ 5. $f(x)=\log_3(2-\sqrt{5-x})$ 6. $f(x)=\ln(x^2-9)$

Answers:

I. 1. 6 2. –2 3. $\frac{1}{3}$ 4. 8 5. $\sqrt{2}$ 6. 2 7. 15

II. 1. e 2. –2 3. $\frac{1}{3}$ 4. 64 5. 33 6. $2\frac{1}{2}$

III. 1. $\log_2 5$ *or* $\frac{\ln 5}{\ln 2}$ 2. 1 3. ±2 4. $\frac{\ln(.01)}{-.05}$ *or* $20\ln(100)$

5. 4 6. 4 7. $\left(2^{\frac{1}{12}}-1\right)$ 8. $\frac{1}{e}$ 9. $\frac{e^3+1}{2}$ 10. $\frac{4+\ln 2}{3}$

11. $\frac{\ln 3}{\ln 2}+5$ *or* $(\log_2 3)+5$ 12. $\frac{e}{2}$ 13. $3e$

14. $\frac{9+\sqrt{81+4e^3}}{2}$ (note: $\frac{9-\sqrt{81+4e^3}}{2}<0$, so it is not in the domain)

15. 20 16. 80 17. 25 18. –3 19. e 20. e^e

21. $-5\frac{1}{2}$ 22. 4 23. 3 24. 2 (note: -3 is not in domain)

25. $\log_7 2$ (or $\frac{\ln 2}{\ln 7}$) 26. $\ln 2$ 27. 0

IV. Solution 1: $\log_a b = \frac{\ln b}{\ln a}$ and $\log_b a = \frac{\ln a}{\ln b}$.

So, $(\log_a b)(\log_b a) = \frac{\ln b}{\ln a} \cdot \frac{\ln a}{\ln b} = 1$

Solution 2: $\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$.

So, $(\log_a b)(\log_b a) = \frac{1}{\log_b a} \cdot \frac{\log_b a}{1} = 1$

V. 1. $\ln\left(22\frac{1}{2}\right)$ 2. $\log_4\left(\frac{x\sqrt[5]{x-y}}{x^2}\right)$

VI. 1. $\ln 10 + 8\ln c - \frac{1}{4}\ln a - \frac{3}{4}\ln b$ 2. $\frac{1}{2}\log_2 a + \frac{1}{2}\log_2(b^2 + c^2)$

VII. 1. $(-7, \infty)$ 2. $(3, 10)$ 3. $(-5, -4) \cup (-4, \infty)$

4. Empty set 5. $(1, 5]$ 6. $(-\infty, -3) \cup (3, \infty)$