PART 1

1. Evaluate the following limits:

$\lim_{x\to 3}\frac{4x+1}{x^2-5}$ $\lim_{x\to -2}\frac{x^2-x-6}{3x+6}$ $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$ $\lim_{x\to 4}\left(x^2+3x-5\right)$

$\lim_{x\to 2}\frac{x^2+x-6}{x-2}$ $\lim_{x\to 1}5$ $\lim_{x\to 2^-}\frac{x^2+4}{x+4}$ $\lim_{x\to -1}\frac{x^2-1}{x+1}$ $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

2. $f(x)=\begin{cases} x^2-3 & \text{if } x<-1 \\ 2x & \text{if } -1<x<4 \\ x^2+1 & \text{if } x\geq 4 \end{cases}$ Evaluate the following:

(a) $f(4)$ (b) $\lim_{x\to 4^-} f(x)$ (c) $\lim_{x\to 4^+} f(x)$ (d) $\lim_{x\to 4} f(x)$

(e) $f(-1)$ (f) $\lim_{x\to -1^-} f(x)$ (g) $\lim_{x\to -1^+} f(x)$ (h) $\lim_{x\to -1} f(x)$

3. Find $f'(x)$ for each of the following functions. DO NOT SIMPLIFY.

(a) $f(x)=4x^5-2x^3+5x-7$ (b) $f(x)=(4x^2+35x-87)(x^5-7x^3+x)$

(c) $f(x)=\frac{1}{(-x^7+3x^2+2)^{20}}$ (d) $f(x)=\frac{x^2+3x-5}{5x^4+3}$ (e) $f(x)=\sqrt{x+(2x+16)^5}$

(f) $f(x)=3x^4-5x^2+9$ (g) $f(x)=(x^3+6x-4)(x^4-x^2+3)$ (h) $f(x)=\frac{1}{3x+2}$

(i) $f(x)=2x^2+3x+4+\sqrt{2x}+\frac{1}{x^2}$ (j) $f(x)=(3x^2+\sqrt{4x-1})^7$

(k) $f(x)=(x^2-\sqrt{x})(7+2x-9x^2)$ (l) $f(x)=\frac{(x^3+5x+1)(x^2+4x+4)}{x^5+3}$

Find $\frac{dh}{dy}$ where $h=\frac{1+y}{2y-5}$

Find $\frac{dy}{dx}\Big|_{x=2}$ where $y=\frac{1}{\sqrt{x^3}}$

Find $\frac{dt}{dx}$ where $t=2y+3$ and $y=2x^2-4x-2$

4. Given that $\frac{d}{dx}(\arctan x) = \frac{1}{x^2+1}$, find the derivative for $f(x) = \arctan(x^7 + 2x + 5)$. DO NOT SIMPLIFY. Note: "arctan x" is the inverse tangent function....but you don't need to be familiar with it to find the derivative of f.

5. Find $\frac{d^2y}{dx^2}$ for $y = \frac{3}{x^2} + \frac{x^2}{3} + \sqrt[3]{x^2} + 3^2$

Suppose $f(x) = \frac{x^3}{x^2+1}$. Find $f'(x)$ and $f''(x)$.

6. Find the EQUATION of the line tangent to the curve at the given point.

(a) $f(x) = \sqrt{x}$ at $x = 4$ (b) $g(x) = x^2 - 3x - 2$ at $x = 3$

7. While studying for your calculus test you are distracted by a squirrel outside the window. It is running back and forth on a power line attached to the library. The position of the squirrel (distance measured in feet from the library) at time t (measured in seconds after you first saw the rodent) is given by $\chi(t) = t^3 - 8t^2 + 6$.
Answers below should have units of measure attached.
(a) How far is the squirrel from the library when you first see it?
(b) What is the instantaneous velocity of the squirrel 5 seconds later?
(c) What is the average velocity of the squirrel over the first five seconds that you observe it?

8. Given cost equation $C(x) = 250 + 4x$ and demand equation $x(p) = 150 - p$,

(a) What is the Revenue equation?

(b) What is the Profit equation?

(c) What is the Marginal Profit equation?

(d) At what price should you sell your product ? (Justify your answer)

9. $f(x) = \sqrt{9 - x^2}$ is the equation of a semicircle. It is the top half of the circle whose center is the origin and whose radius is 3 units (see picture). Suppose that (x_0, y_0) is a point on the semicircle but not $(0, 3)$, $(-3, 0)$ or $(3, 0)$. Show that the line tangent to the semicircle at (x_0, y_0) is perpendicular to the line that goes through (x_0, y_0) and the origin.