PART 2

1. Evaluate the following limits. You do not have to show work, but if your answer is not correct and no work is shown there can be no possibility of partial credit.

$$\lim_{x\to\infty}\frac{3x^4-2x^2+1}{-5x^4+x^3-7} \qquad \lim_{x\to-\infty}\frac{\sqrt{x^4+6}}{x^3+1} \qquad \lim_{x\to2^+}\frac{x^2-7}{5x-10} \qquad \lim_{x\to3^-}\frac{2x^2-5x-3}{x-3}$$

$$\lim_{x\to-\infty} 3x^4+6x^3-5x^2-1 \qquad \lim_{x\to5}\frac{x^2-25}{\sqrt{x}-\sqrt{5}} \qquad \lim_{x\to-3}\frac{x^2+2}{x^2+6x+9}$$

$$\lim_{x\to\infty}\frac{4x^3+2x^2-3}{3x^4+6x+9} \qquad \lim_{x\to-\infty}\frac{3x^3+2x}{\sqrt{5x^6+6x^5+7x+1}} \qquad \lim_{x\to-\infty}\frac{x^7+2x^5-5}{3x^5+x^4+2x}$$

2.

$$f(x)=\frac{-3x^3}{x(x-2)^2}$$

(a) What is the domain of f ?

(b) Write, and evaluate, appropriate limits to enable you to determine the existence, or not, of any horizontal or vertical asymptotes, and the behavior of the graph near those asymptotes.

(c) Use the information from parts (a) and (b) above to draw a rough sketch of the graph of f. You do not have to use derivatives to refine your graph.

3. $f(x)=\dfrac{4x}{3x^2-10x}$

(a) What is the domain of f?

(b) Where is $f(x)$ positive? Negative?

(c) Give the equations of all vertical and horizontal asymptotes of f.

4. Find the intervals where $f(x)=x^4-4x^3-8x^2$ is increasing and decreasing. Find all local extrema for f.

5. Find the intervals where $f(x)=\frac{1}{20}x^5-\frac{1}{6}x^4+x$ is concave up and concave down. Find all inflection points of f.

6. Find the absolute maximum and minimum values of $f(x)=x^3-9x$ on the interval $[-1,\ 3]$.

7. For each function below, find the following information. If a particular item does not apply, write "none." Then use the information to graph the function. You must show sufficient work to justify all answers.

Interval(s) where f is decreasing. ____________________
Interval(s) where f is increasing.____________________
Intervals(s) where f is concave up.____________________
Intervals(s) where f is concave down.____________________
f has local minimum(s) at ____________________
f has local maximum(s) at ____________________
f has point(s) of inflection ____________________(give coordinates)
f has absolute minimum value of __________at__________
f has absolute maximum value of __________at__________
Horizontal Asymptotes ____________________(give equations)
Vertical Asymptotes____________________(give equations)

$f(x) = x - 3x^{\frac{2}{3}}$ *where* $x \geq -1$ $\qquad f(x) = x^3 - 27x + 8$ *on* $[-2,6]$ $\qquad f(x) = \dfrac{2x^2}{3+x^2}$

8. Given $f(x) = \dfrac{x^2}{x^2-4}$ and $f'(x) = \dfrac{-8x}{(x^2-4)^2}$ and $f''(x) = \dfrac{8(3x^2+4)}{(x^2-4)^3}$
   (a) Find the domain of f.
   (b) Find the roots of f.
   (c) Find the intervals where the graph of f is above/below the x-axis.
   (d) What are the vertical asymptotes of f?
   (e) What are the horizontal asymptotes of f?
   (f) Find the intervals where f is increasing/decreasing.
   (g) Find any local maxima/minima.
   (h) What are the inflection points of f?
   (i) Find the intervals where f is concave up/concave down.
   (j) Use the information from the previous parts to sketch f.

9. The expected profit to be realized from the sale of x items is given by the formula $P(x) = -x^3 - 18x^2 + 480x + 17$. Determine the number of items that should be sold to maximize profit.

10. A farmer has bought 1200 feet of fencing and wants to build a rectangular fence with two dividers. The dividers run parallel to each other and to two sides of the rectangle. What are the dimensions of the rectangle that maximizes the total area fenced in? Make sure to check that your answer actually is a maximum.

11. A community service organization has \$6,400 to spend on fencing for a rectangular playground. They want to put fancy fencing on the front and cheaper fencing on the back and sides. Fancy fencing costs \$6 per linear foot. Cheaper fencing costs \$2 per linear foot. What are the dimensions of the largest area that can be fenced?

12. For each function below, find the point that satisfies the Mean Value Theorem on the indicated interval.

$f(x) = \dfrac{x+3}{x-2}$ on the interval [3,7] $\qquad$ $f(x) = \sqrt[3]{x}$ on the interval [0, 1].

13. The Goodday Tire Company has determined that the daily demand, q, for their high-tech tires is related to the unit price p (in dollars) by the equation $q = \sqrt{144 - p}$. They also know that the cost to produce these tires is $C(q) = 36q + 64$.

(a) Find the elasticity function *E(p)*. Simplify your answer as much as possible.

(b) The current price per tire is $100. If this price is decreased slightly, will the Revenue increase or decrease? Justify your answer.

(c) Write an equation to express the Profit as a function of the demand q.

(d) What quantity sold will yield the maximum Profit? Show work. Be sure to justify that your value of q will yield the absolute maximum profit.

14. For each of the following two demand functions, find the elasticity function *E(p)*. Then find the range of values where the demand is elastic and where it is inelastic.

(a) $q = 30 - p^2$ $\qquad$ (b) $q = \sqrt{24 - 3p^2}$

15. The quantity demanded for an item given price p is given by the function $q = 200 - 4p$.
(a) Find the elasticity function $E(p)$.
(b) For the sale price $p = 10$ determine if the demand is elastic or inelastic. What happens to revenue if you increase price slightly?
(c) Using E, determine what sale price will yield maximum revenue.