Math 220 Section 18 Supplementary Exercises

For each of the two functions below, find the absolute extrema (max and/or min). Use calculus techniques (first and second derivatives and limits) to graph the function.

1. $f(x) = \dfrac{(x+1)^2}{x^2+1}$ 2. $g(x) = x^2 - |x|$

(If you are stuck on Problem 2, there is a hint posted after the solution to Problem 1.)

Answers:

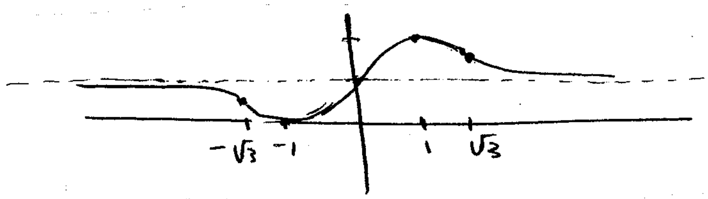
1. Absolute maximum value of 2 occurs at $x = 1$. (also a local max)
 Absolute minimum value of 0 occurs at $x = -1$. (also a local min)

 Solution aids: $f'(x) = \dfrac{2-2x^2}{(x^2+1)^2} = \dfrac{2(1-x)(1+x)}{(x^2+1)^2}$ Critical pts: $x = -1$, $x = 1$

 $f''(x) = \dfrac{4x(x^2-3)}{(x^2+1)^3}$ so, $f''(x) = 0$ at $x = 0$, $x = \sqrt{3}$, $x = -\sqrt{3}$

 Points of Inflection are: $(0, 1)$, $\left(\sqrt{3}, 1+\frac{\sqrt{3}}{2}\right)$, $\left(-\sqrt{3}, 1-\frac{\sqrt{3}}{2}\right)$

 $\lim_{x\to-\infty} f(x) = 1$ and $\lim_{x\to\infty} f(x) = 1$ so, horizontal asymptote $y = 1$



Hint for Problem 2: Rewrite g into the equivalent form $g(x) = \begin{cases} x^2 + x & x < 0 \\ x^2 - x & x \ge 0 \end{cases}$

2. Absolute minimum value of $-\frac{1}{4}$ occurs at $x = -\frac{1}{2}$ and at $x = \frac{1}{2}$.
 There is no Absolute maximum. There is a local max at $(0, 0)$.

 Solution aids: $g'(x) = \begin{cases} 2x+1 & x < 0 \\ 2x-1 & x > 0 \end{cases}$ Critical pts.: $x = -\frac{1}{2}$, $x = \frac{1}{2}$, $x = 0$

 $g'(x) = 0$ at $x = -\frac{1}{2}$ and at $x = \frac{1}{2}$ AND $g'(x)$ D.N.E. at $x = 0$ (LHD$\neq$RHD)

 $g''(x) = \begin{cases} 2 & x < 0 \\ 2 & x > 0 \end{cases}$ $g''(x) = 0$ never, $g''(x)$ D.N.E. at $x = 0$ (because $g'(0)$ D.N.E.)

 So, there are no points of inflection. g is always concave up.

 $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$