

Supplementary Exercises – Chapter 19 (Answers Below)

Differentiate. Do NOT simplify.

1. $f(x) = e^{\sqrt{x}}$

2. $f(x) = x^2 e^x$

3. $f(x) = \sqrt{1 - 2e^x}$

4. $f(x) = e^{-\frac{1}{x}}$

5. $f(x) = \frac{e^{3x}}{1 + e^x}$

6. $f(x) = xe^{-x^2}$

7. $f(x) = \frac{e^x}{x}$

8. $f(x) = \sqrt[3]{2x + e^{2x}}$

9. $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

10. $f(x) = e^x - x^e$

11. $f(x) = (2x - 5e^x)^3$

12. $f(x) = e^{e^{2x}}$

13. Given $g(x) = e^{-2x}$. Find:

(a) $g'(x)$ (b) $g''(x)$ (c) $g^{(8)}(x)$ (d) $g^{(n)}(x)$ for n a positive integer

14. Show that $y = e^{2x} + e^{-3x}$ satisfies the equation $y'' + y' - 6y = 0$.

15. Find the equation of the line tangent to the curve $y = x^2 e^{-x}$ where $x = 1$.

16. Find the limits:

(a) $\lim_{x \rightarrow \infty} e^{\frac{x^2 + 4x + 2}{4x^2 + 5x + 1}}$

(b) $\lim_{x \rightarrow 2^+} 10^{\frac{5}{x-2}}$

(c) $\lim_{x \rightarrow 2^-} 10^{\frac{5}{x-2}}$

(d) $\lim_{x \rightarrow -\frac{1}{2}} 2^{6x+1}$

(e) $\lim_{x \rightarrow -\infty} e^{\frac{x}{\sqrt{4x^2 + x + 3}} + \frac{3}{2}}$

(f) $\lim_{x \rightarrow \infty} e^{\frac{x}{\sqrt{4x^2 + x + 3}} + \frac{3}{2}}$

Problem 17 can be done after studying Chapters 13 and 14:

17. For $f(x) = xe^x$, find the interval(s) where f is increasing/decreasing, the interval(s) where f is concave up/down, and any points of inflection.

Problem 18 can be done after studying Chapter 18:

18. Find the absolute maximum value of the function $f(x) = x - e^x$.

Answers:

1. $e^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}} \right)$ 2. $2xe^x + e^x x^2$ 3. $\frac{1}{2}(1-2e^x)^{-\frac{1}{2}}(-2e^x)$ 4. $e^{-\frac{1}{x}} \left(\frac{1}{x^2} \right)$

5. $\frac{e^{3x} \cdot 3(1+e^x) - e^x \cdot e^{3x}}{(1+e^x)^2}$ 6. $e^{-x^2} + e^{-x^2}(-2x)x$ 7. $\frac{e^x x - e^x}{x^2}$

8. $\frac{1}{3}(2x + e^{2x})^{-\frac{2}{3}}(2 + e^{2x} \cdot 2)$ 9. $\frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$

10. $e^x \cdot e \cdot x^{e-1}$ 11. $3(2x - 5e^x)^2(2 - 5e^x)$ 12. $e^{e^x} \cdot e^{e^x} \cdot e^x$

13. (a) $e^{-2x}(-2)$ (b) $e^{-2x}(-2)(-2)$ (c) $e^{-2x}(-2)^8$ (d) $e^{-2x}(-2)^n$

14. $\left. \begin{array}{l} y = e^{2x} + e^{-3x} \\ y' = 2e^{2x} - 3e^{-3x} \\ y'' = 4e^{2x} + 9e^{-3x} \end{array} \right\} \text{so, } \left\{ \begin{array}{l} y'' + y' - 6y \\ = (4e^{2x} + 9e^{-3x}) + (2e^{2x} - 3e^{-3x}) - 6(e^{2x} + e^{-3x}) \\ = 6e^{2x} - 6e^{2x} + 6e^{-3x} - 6e^{-3x} \\ = 0 \end{array} \right.$

15. $y - \frac{1}{e} = \frac{1}{e}(x-1)$

16. (a) $\sqrt[4]{e}$ (b) ∞ (c) 0 (d) $\frac{1}{4}$ (e) e (f) e^2

17. Increasing on $(-1, \infty)$, decreasing on $(-\infty, -1)$

Concave up on $(-2, \infty)$, concave down on $(-\infty, -2)$. Point of Inflection: $\left(-2, \frac{-2}{e^2}\right)$

18. Absolute maximum value of -1 at $x=0$.