

Supplementary Exercises – Chapter 20 (answers below)

Differentiate.

1.  $f(x) = \ln(x^3 - 2x + 3)$

2.  $f(x) = \frac{\ln x}{x}$

3.  $f(x) = x \ln x - x$

4.  $f(x) = \ln(\ln x)$

5.  $f(x) = \ln\left(\frac{x+1}{x^3-x}\right)$

6.  $f(x) = e^{(\ln x)^2}$

Differentiate. Do NOT simplify.

7.  $f(x) = 3^x$

8.  $f(x) = 3^{6x^2+2x+1}$

9.  $f(x) = (\sqrt{2})^x + x^{\sqrt{2}}$

10.  $f(x) = \log_5 x$

11.  $f(x) = \log_{10}(5x^3 + x^2)$

12.  $f(x) = \log_{10}(3^x)$

13.  $f(x) = x^x$

(hint for #13: rewrite  $x^x$  using base  $e$ )

Find the equation of the line tangent to the curve at the given point.

14.  $f(x) = 4^x$       $x = 3$

15.  $f(x) = \ln(8 - 4x)$       $x = 1$

16.  $f(x) = \log_2\left(x + \frac{1}{x}\right)$       $x = 1$

Evaluate the following limits:

17.  $\lim_{x \rightarrow 4} \ln(3x - 4)$

18.  $\lim_{x \rightarrow \infty} [\ln(2 + x) - \ln(1 + x)]$

19.  $\lim_{x \rightarrow 5^+} \log_3(x^2 - 25)$

20. Suppose you invest \$ 5,000 at an annual rate of 5%, compounded continuously. How long will it take for your investment to reach \$ 20,000?

Problems 21 and 22 can be done after studying Chapters 12 and 14:

Find the local extrema for  $f$ . Use the Second Derivative Test to verify local maximum or local minimum.

21.  $f(x) = x - \ln x$

22.  $f(x) = \frac{\ln x}{x}$

Answers:

1.  $\frac{3x^2 - 2}{x^3 - 2x + 3}$

2.  $\frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

3.  $\ln x + \frac{1}{x} \cdot x - 1 = \ln x$

4.  $\frac{1}{\ln x} \cdot \frac{1}{x}$

5.  $\left(\frac{x^3 - x}{x + 1}\right) \cdot \left(\frac{1 \cdot (x^3 - x) - (3x^2 - 1)(x + 1)}{(x^3 - x)^2}\right)$

6.  $e^{(\ln x)^2} (2 \ln x) \cdot \frac{1}{x}$

7.  $3^x \ln 3$

8.  $3^{6x^2 + 2x + 1} (\ln 3)(12x + 2)$

9.  $(\sqrt{2})^x (\ln \sqrt{2}) + \sqrt{2} x^{\sqrt{2}-1}$

10.  $\frac{1}{x \ln 5}$

11.  $\frac{15x^2 + 2x}{(5x^3 + x^2) \ln 10}$

12.  $\frac{1}{3^x \ln 10} \cdot 3^x \ln 3 = \frac{\ln 3}{\ln 10}$

13.  $x^x (\ln x + 1)$

14.  $y - 64 = 64 \ln 4 (x - 3)$

15.  $y - \ln 4 = -(x - 1)$

16.  $y = 1$

17.  $\ln 8$

18.  $0$

19.  $-\infty$

20.  $20(\ln 4)$  years...approx. 27.7 years

21. local MINIMUM at  $x = 1$

22. Local MAXIMUM at  $x = e$