

Math 220 -- Logarithm Review and Practice Sheet (Answers Below)

THESE EXERCISES ARE TO BE DONE WITHOUT A CALCULATOR.

I. Find the exact value of each expression:

1. $\log_2 64$ 2. $\log_6 \frac{1}{36}$ 3. $\log_8 2$ 4. $e^{3\ln 2}$
5. $\ln e^{\sqrt{2}}$ 6. $\log_5 10 + \log_5 20 - 3\log_5 2$ 7. $2^{(\log_2 3 + \log_2 5)}$

II. Find x :

1. $\log_x e^5 = 5$ 2. $\log_{\frac{1}{4}} 16 = x$ 3. $\log_x 9 = -2$
4. $\log_{16} x = \frac{3}{2}$ 5. $10^{\log_{10} x} = 33$

III. Solve for x . Do not use a calculator (an answer like $\frac{\ln 2}{7}$ is OK).

1. $25 = 5(2^x)$ 2. $\ln x = 0$ 3. $\ln e^{x^2} = 4$ 4. $.01 = e^{-.05x}$
5. $e^{\ln 4x} = 16$ 6. $\ln e^{3x} = 12$ 7. $2 = (1+x)^{12}$ 8. $\ln x = -1$
9. $\ln(2x-1) = 3$ 10. $e^{3x-4} = 2$ 11. $2^{x-5} = 3$ 12. $\ln x + \ln(4x) = 2$
13. $\ln(3x^2) - \ln(9x) = 1$ 14. $\ln(x-9) + \ln x = 3$ 15. $\log_{10} 5 + \log_{10} x = 2$
16. $\log_{10} x - \log_{10} 8 = 1$ 17. $\log_4 x = \log_2 5$ 18. $\frac{\ln 1}{\ln 5} = x + 3$
19. $\ln(\ln x) = 0$ 20. $\ln(\ln x) = 1$

IV. Show that $(\log_a b)(\log_b a) = 1$ for $a, b > 0$; $a, b \neq 1$

Answers:

- I. 1. 6 2. -2 3. $\frac{1}{3}$ 4. 8 5. $\sqrt{2}$ 6. 2 7. 15
II. 1. e 2. -2 3. $\frac{1}{3}$ 4. 64 5. 33
III. 1. $\log_2 5$ or $\frac{\ln 5}{\ln 2}$ 2. 1 3. ± 2 4. $\frac{\ln(.01)}{-.05}$ or $20\ln(100)$
5. 4 6. 4 7. $\left(2^{\frac{1}{12}} - 1\right)$ 8. $\frac{1}{e}$ 9. $\frac{e^3+1}{2}$ 10. $\frac{4+\ln 2}{3}$
11. $\frac{\ln 3}{\ln 2} + 5$ or $(\log_2 3) + 5$ 12. $\frac{e}{2}$ 13. $3e$

14. $\frac{9 + \sqrt{81 + 4e^3}}{2}$ (note: $\frac{9 - \sqrt{81 + 4e^3}}{2} < 0$, so it is not in the domain)

15. 20

16. 80

17. 25

18. -3

19. e

20. e^e

IV.

Solution 1: $\log_a b = \frac{\ln b}{\ln a}$ and $\log_b a = \frac{\ln a}{\ln b}$.

$$\text{So, } (\log_a b)(\log_b a) = \frac{\ln b}{\ln a} \cdot \frac{\ln a}{\ln b} = 1$$

Solution 2: $\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$.

$$\text{So, } (\log_a b)(\log_b a) = \frac{1}{\log_b a} \cdot \frac{\log_b a}{1} = 1$$