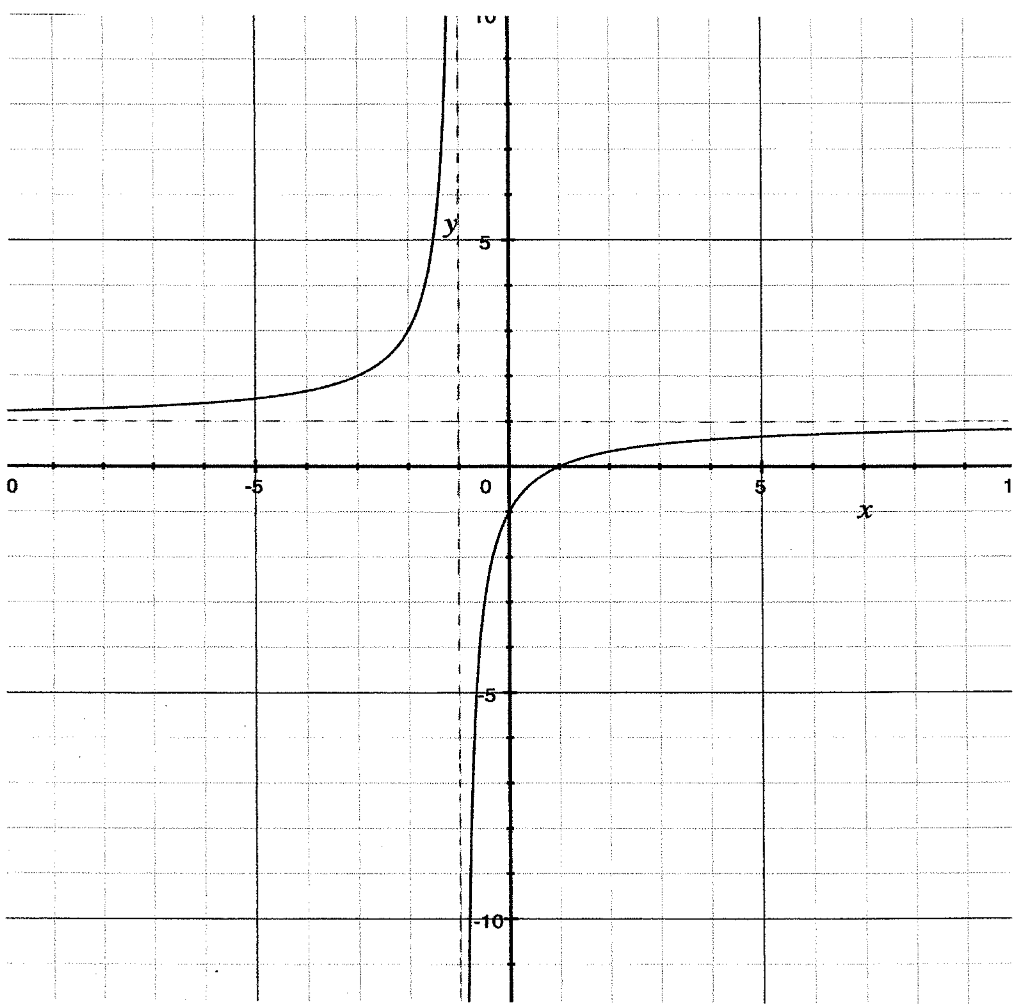
Text Answers Section 17

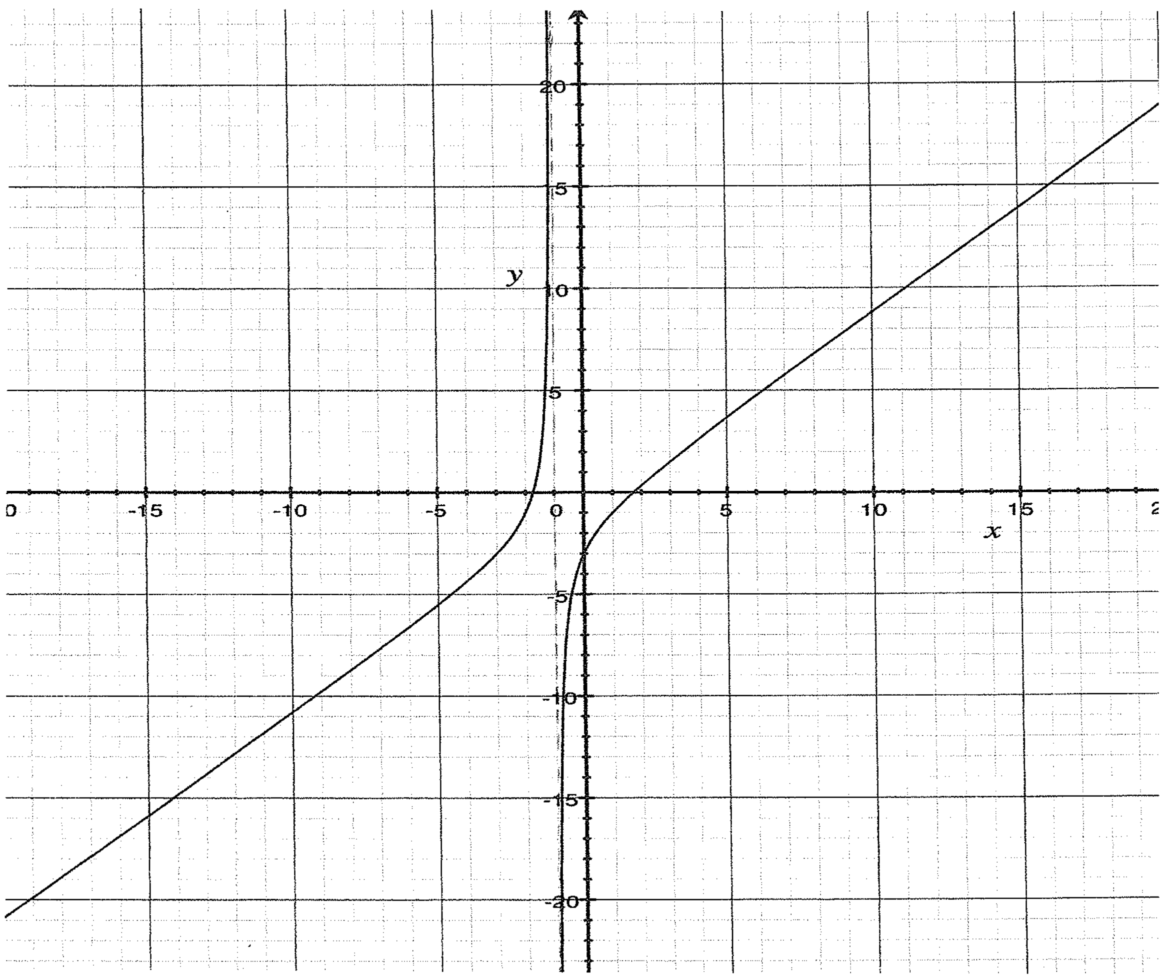
1. $f(x)=\dfrac{x-1}{x+1}$

Domain $x \neq -1$	y-intercept $(0,-1)$	x- intercept $x=1$
$f'(x)=\dfrac{2}{(x+1)^2}$	$f'=0$ nowhere	f' DNE at $x=-1$
Incr. $(-\infty,-1)\cup(-1,\ \infty)$	Decr. nowhere	
Loc min none	Loc max at none	
$f''(x)=\dfrac{-4}{(x+1)^3}$	$f''=0$ nowhere	f'' DNE at $x=-1$
Conc. UP $(-\infty,-1)$	Conc. DOWN $(-1,\ \infty)$	P.O.I. none
Horiz. Asymp. $y=1$	Vertical Asymp. $x=-1$	



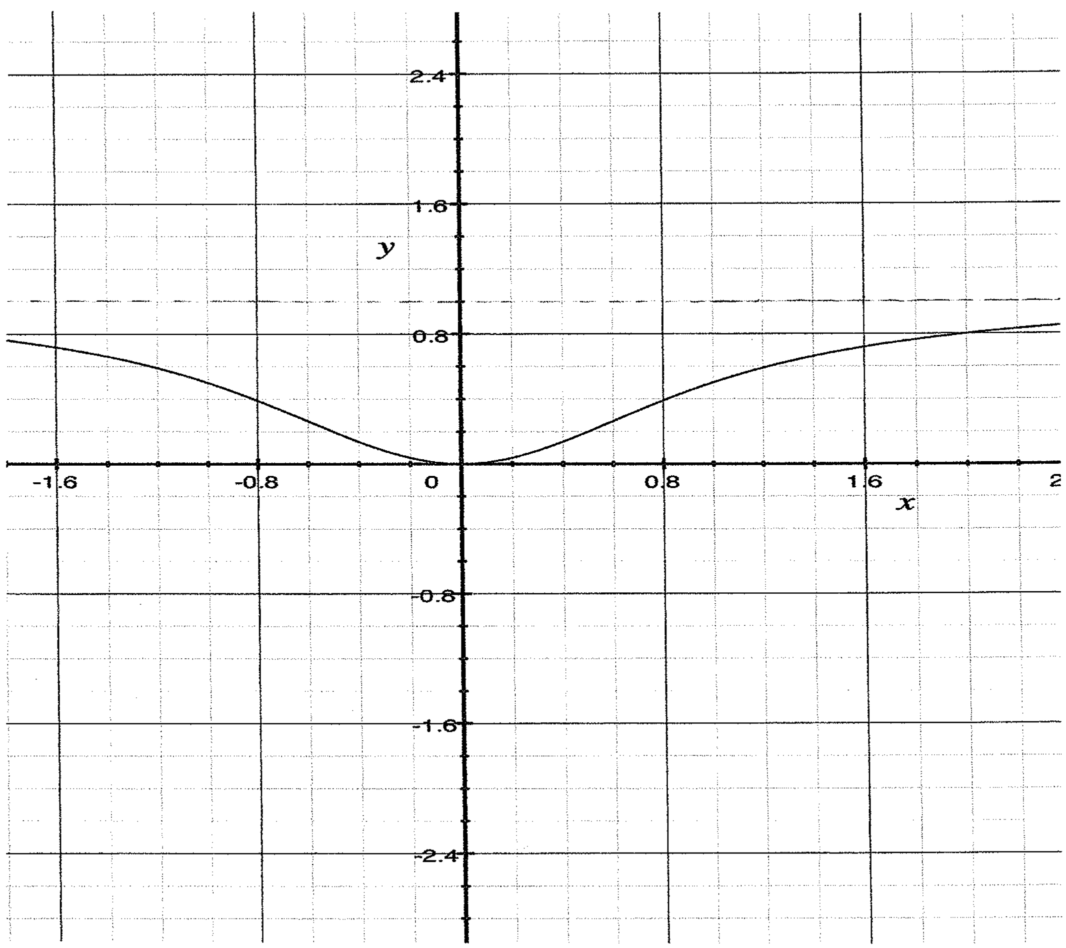
2. $f(x)=\dfrac{x^2-3}{x+1}$

Domain $x \neq -1$	y-intercept $(0, -3)$	x- intercepts $x=\pm\sqrt{3}\approx\pm 1.7$
$f'(x)=\dfrac{x^2+2x+3}{(x+1)^2}$	$f'=0$ nowhere	f' *DNE* at $x=-1$
Incr. $(-\infty,-1)\cup(-1,\infty)$	Decr. nowhere	
Loc min none	Loc max at none	
$f''(x)=\dfrac{-4}{(x+1)^3}$	$f''=0$ nowhere	f'' *DNE* at $x=-1$
Conc. UP $(-\infty,-1)$	Conc. DOWN $(-1,\infty)$	P.O.I. none
Horiz. Asymp. none	Vertical Asymp. $x=-1$	



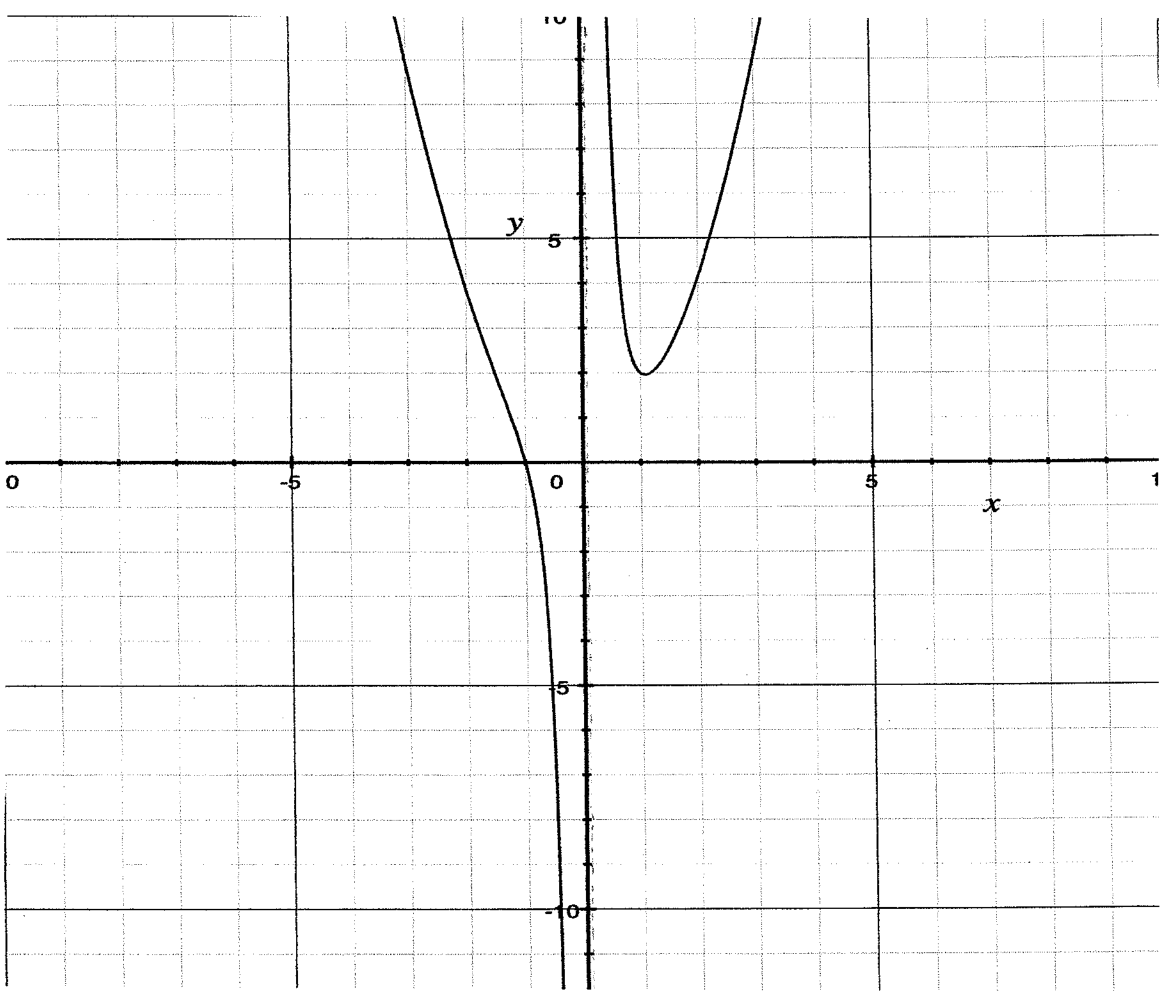
3. $f(t)=\dfrac{t^2}{1+t^2}$

Domain $(-\infty,\ \infty)$	y-intercept $(0,\ 0)$	x- intercept $x=0$
$f'(t)=\dfrac{2t}{(1+t^2)^2}$	$f'=0$ at $x=0$	f' *DNE* never
Incr. $(0,\ \infty)$	Decr. $(-\infty,\ 0)$	
Loc min at $x=0$	Loc max none	
$f''(x)=\dfrac{-2(3t^2-1)}{(1+t^2)^3}$	$f''=0$ at $x=\pm\sqrt{\frac{1}{3}}\approx\pm.57$	f'' *DNE* never
Conc. UP $\left(-\sqrt{\frac{1}{3}},\ \sqrt{\frac{1}{3}}\right)$	Conc. DOWN $\left(-\infty,\ -\sqrt{\frac{1}{3}}\right)\cup\left(\sqrt{\frac{1}{3}},\ \infty\right)$	P.O.I. $\left(-\sqrt{\frac{1}{3}},\ \frac{1}{4}\right)$ $\left(\sqrt{\frac{1}{3}},\ \frac{1}{4}\right)$
Horiz. Asymp. $y=1$	Vertical Asymp. none	Note: $\sqrt{\frac{1}{3}}\approx.58$



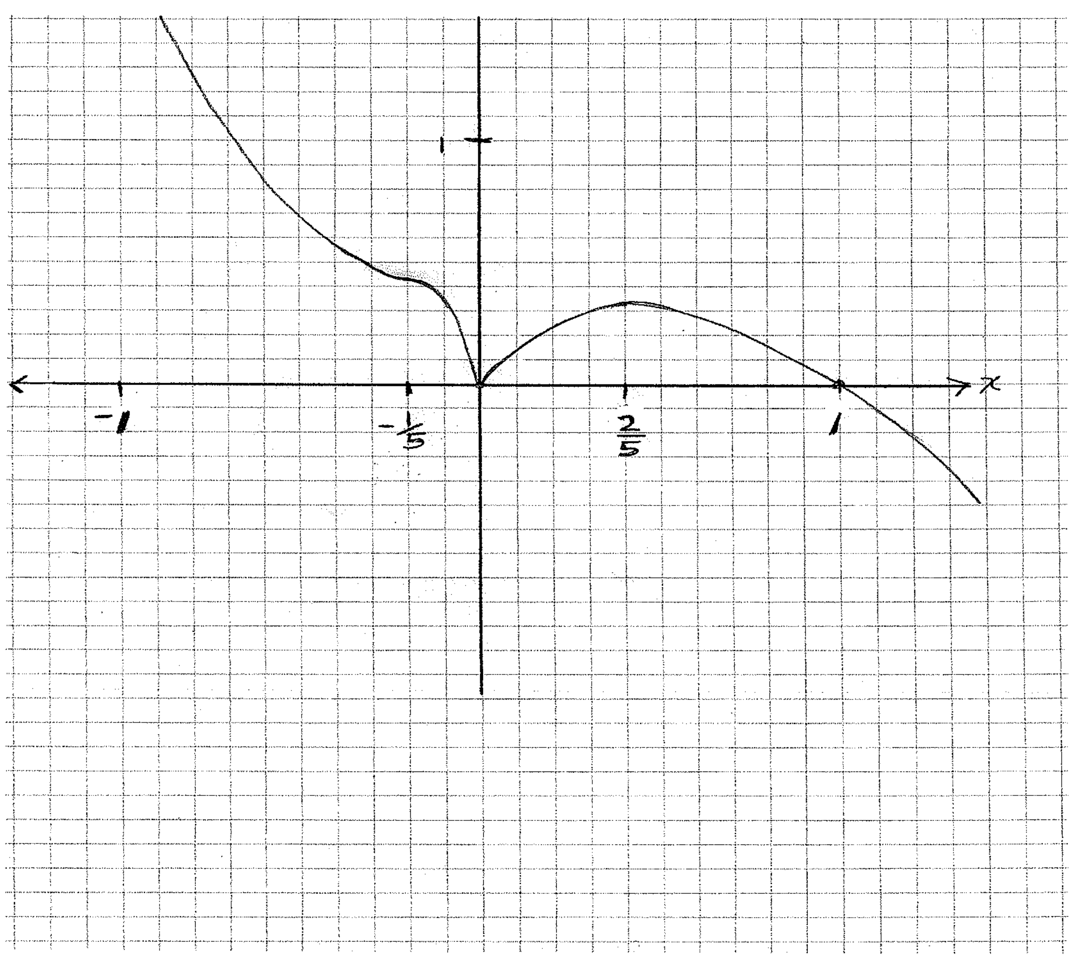
4. $f(x) = x^2 + \frac{1}{x^3} = \frac{x^5+1}{x^3}$

Domain $x \neq 0$	y-intercept none	x- intercept $x = -1$
$f'(x) = 2x - 3x^{-4} = \dfrac{2x^5 - 3}{x^4}$	$f' = 0$ at $x = \sqrt[5]{\frac{3}{2}} \approx 1.1$	f' *DNE* at $x = 0$
Incr. $\left(\sqrt[5]{\frac{3}{2}},\ \infty\right)$	Decr. $(-\infty,\ 0) \cup \left(0,\ \sqrt[5]{\frac{3}{2}}\right)$	
Loc min at $x = \sqrt[5]{\frac{3}{2}}$	Loc max none	
$f''(x) = 2 + 12x^{-5} = \dfrac{2(x^5+6)}{x^5}$	$f'' = 0$ at $x = -\sqrt[5]{6} \approx -1.4$	f'' *DNE* at $x = 0$
Conc. UP $\left(-\infty,\ -\sqrt[5]{6}\right) \cup (0,\ \infty)$	Conc. DOWN $\left(-\sqrt[5]{6},\ 0\right)$	P.O.I. $\left(-\sqrt[5]{6},\ \frac{5}{\sqrt[5]{6^3}}\right)$
Horiz. Asymp. None	Vertical Asymp. $x = 0$	



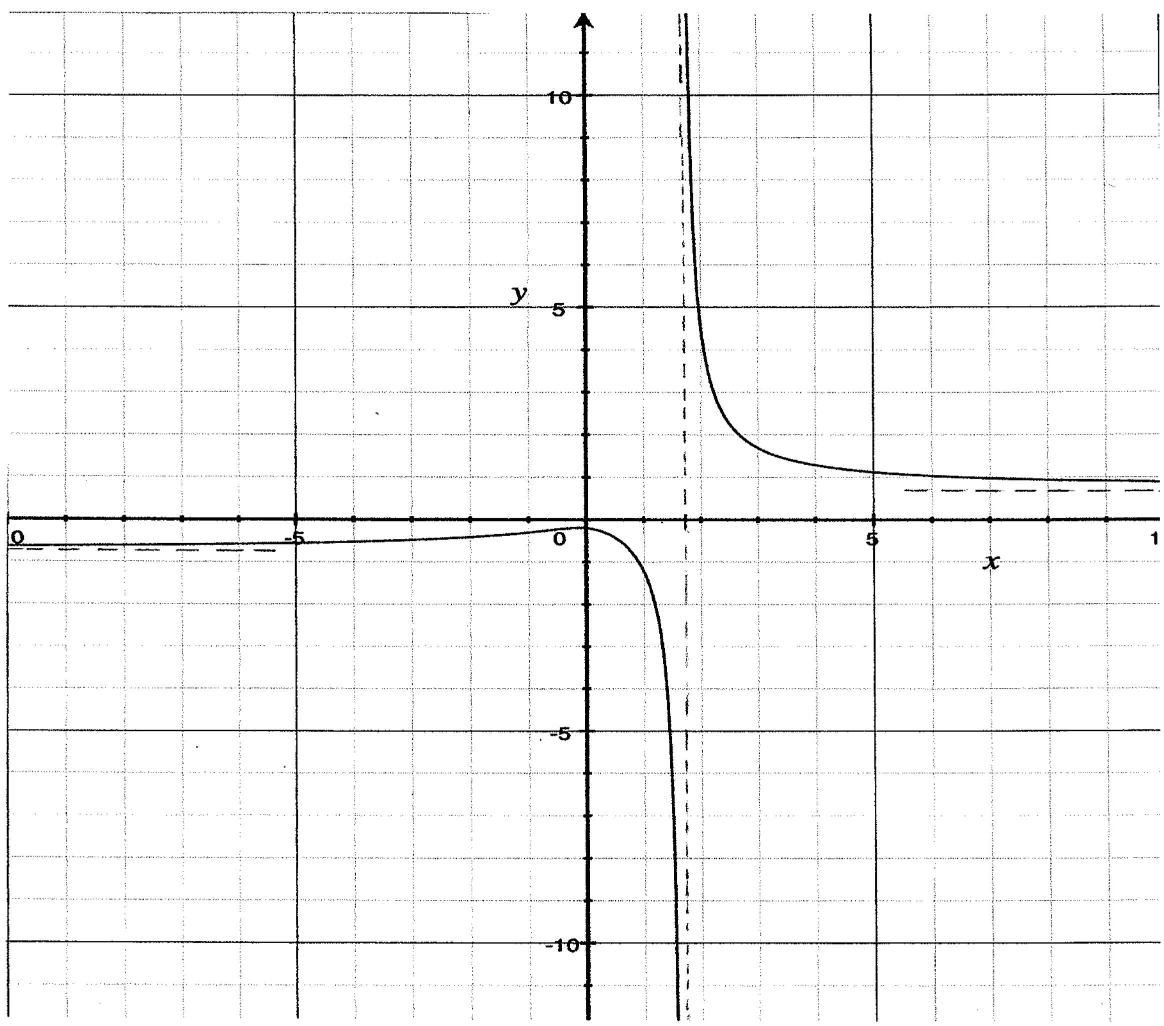
5. $y = x^{\frac{2}{3}} - x^{\frac{5}{3}} = x^{\frac{2}{3}}(1-x)$

Domain $(-\infty,\ \infty)$	y-intercept $(0,\ 0)$	x- intercepts $(0,\ 0)$, $(1,\ 0)$
$y' = \frac{2}{3}x^{-\frac{1}{3}} - \frac{5}{3}x^{\frac{2}{3}} = \frac{1}{3}x^{-\frac{1}{3}}(2-5x)$	$y' = 0$ at $x = \frac{2}{5}$	y' *DNE* at $x = 0$
Incr. $\left(0,\ \frac{2}{5}\right)$	Decr. $(-\infty,\ 0)\cup\left(\frac{2}{5},\ \infty\right)$	
Loc min at $x = 0$	Loc max at $x = \frac{2}{5}$	
$y'' = -\frac{2}{9}x^{-\frac{4}{3}} - \frac{10}{9}x^{-\frac{1}{3}} = -\frac{2}{9}x^{-\frac{4}{3}}(1+5x)$	$y'' = 0$ at $x = -\frac{1}{5}$	y'' *DNE* at $x = 0$
Conc. UP $\left(-\infty,\ -\frac{1}{5}\right)$	Conc. DOWN $\left(-\frac{1}{5},\ \infty\right)$	P.O.I. $\left(-\frac{1}{5},\ \frac{6}{5\sqrt[3]{25}}\right)$
Horiz. Asymp. None	Vertical Asymp. none	



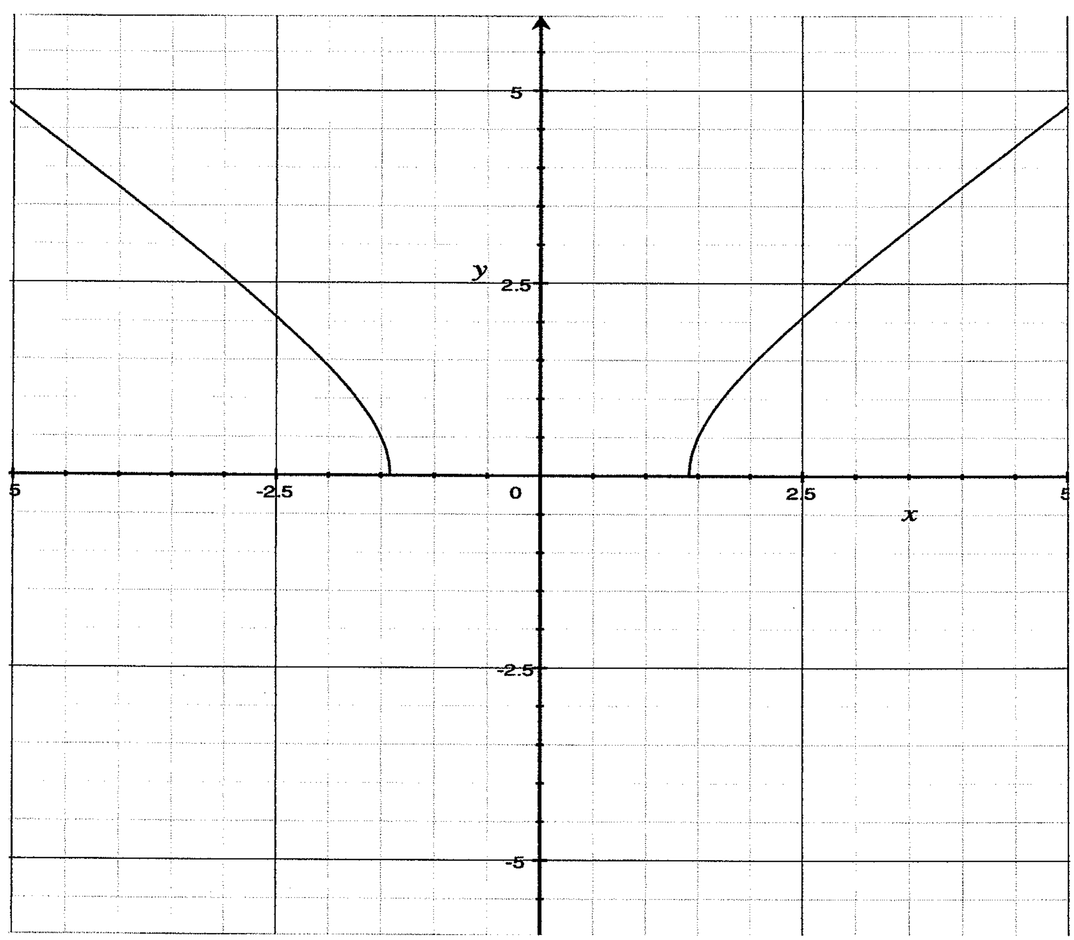
6. $f(x)=\dfrac{\sqrt{5x^2+1}}{3x-5}$

Domain $x \neq \frac{5}{3}$	y-intercept $\left(0, -\frac{1}{5}\right)$	x- intercepts none
$f'(x)=\dfrac{-(25x+3)}{(3x-5)^2\sqrt{5x^2+1}}$	$f'=0$ at $x=-\frac{3}{25}$	f' DNE $x=\frac{5}{3}$
Incr. $\left(-\infty, -\frac{3}{25}\right)$	Decr. $\left(-\frac{3}{25}, \frac{5}{3}\right)\cup\left(\frac{5}{3}, \infty\right)$	
Loc min none	Loc max at $x=-\frac{3}{25}$	
$f''(x)=$ Too complex. Don't bother.		
Horiz. Asymp on Right $y=\frac{\sqrt{5}}{3}\approx .75$	Horiz. Asymp. on Left $y=-\frac{\sqrt{5}}{3}\approx -.75$	Vertical Asymp $x=\frac{5}{3}$



7. $f(x)=\sqrt{x^2-2}=\left(x^2-2\right)^{\frac{1}{2}}$

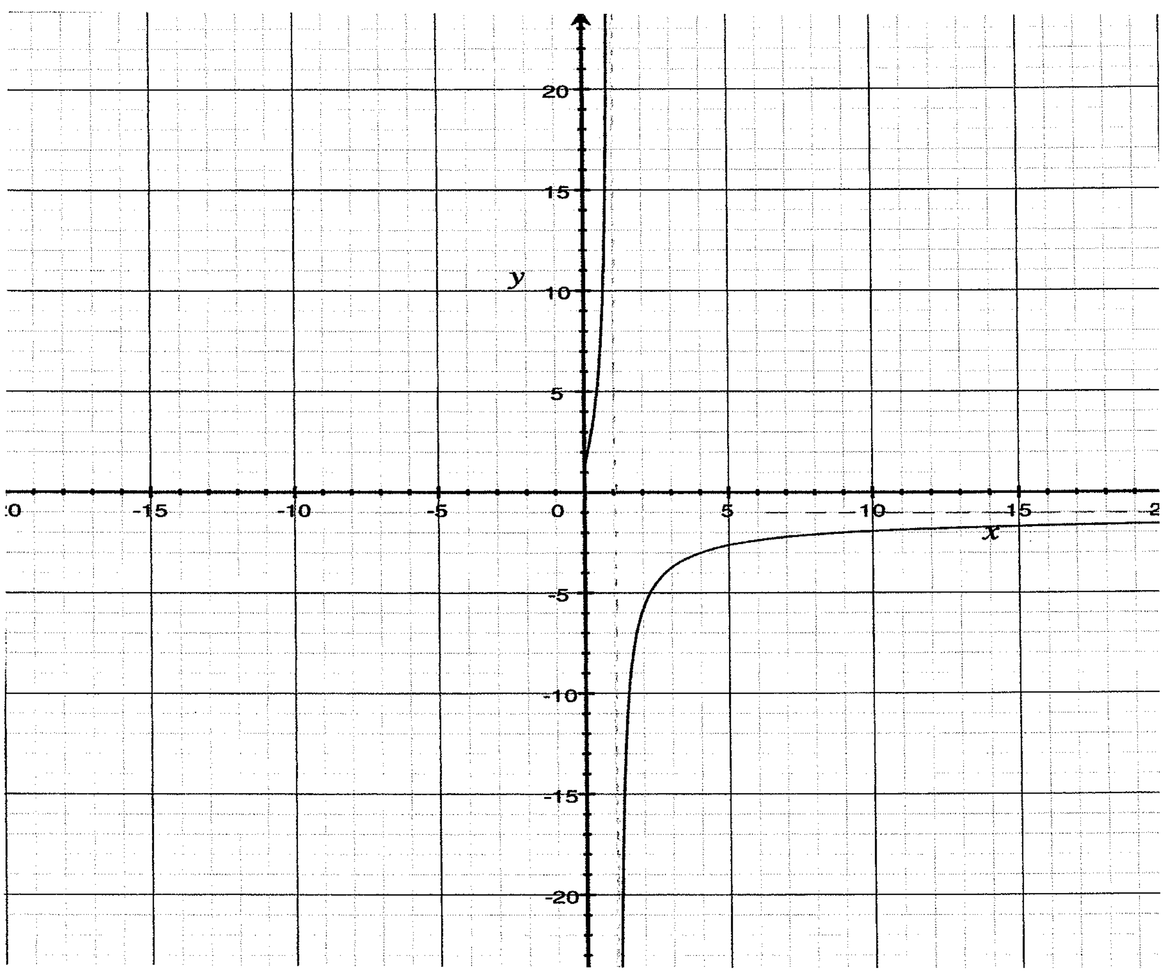
Domain $\left(-\infty,-\sqrt{2}\right]\cup\left[\sqrt{2},\infty\right)$	y-intercept none	x- intercepts $x=\pm\sqrt{2}\approx\pm 1.4$
$f'(x)=\dfrac{x}{\sqrt{x^2-2}}$	$f'=0$ nowhere in domain	f' DNE at $x=\pm\sqrt{2}$
Incr. $\left(\sqrt{2},\infty\right)$	Decr. $\left(-\infty,-\sqrt{2}\right)$	
Loc min none	Loc max none	
$f''(x)=\dfrac{-2}{(x^2-2)^{\frac{3}{2}}}$	$f''=0$ never	f'' DNE at $x=\pm\sqrt{2}$
Conc. UP nowhere	Conc. DOWN $\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$	P.O.I. none
Horiz. Asymp. none	Vertical Asymp. none	



8. $f(x)=\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

Domain $[0,\ 1)\cup(1,\ \infty)$	y-intercept $(0,\ 1)$	x- intercepts none
$f'(x)=\dfrac{1}{\sqrt{x}(1-\sqrt{x})^2}=\dfrac{1}{x^{\frac{1}{2}}+2x+x^{\frac{3}{2}}}$	$f'=0$ nowhere	f' *DNE* at $x=0,\ x=1$
Incr. $(0,\ 1)\cup(1,\ \infty)$	Decr. nowhere	
Loc min none	Loc max none	
$f''(x)=\dfrac{-x^{-\frac{1}{2}}+3}{2x(1-x^{\frac{1}{2}})^3}=\dfrac{-1+3x^{\frac{1}{2}}}{2x^{\frac{3}{2}}(1-x^{\frac{1}{2}})^3}$	$f''=0$ at $x=\frac{1}{9}$	f'' *DNE* at $x=0,\ x=1$
Conc. UP $\left(\frac{1}{9},\ 1\right)$	Conc. DOWN $\left(0,\ \frac{1}{9}\right)\cup(1,\ \infty)$	P.O.I. $\left(\frac{1}{9},\ 2\right)$
Horiz. Asymp. $y=-1$	Vertical Asymp. $x=1$	

Note: Concavity on interval $\left(0,\ \frac{1}{9}\right)$ very very hard to see due to scale on axes.

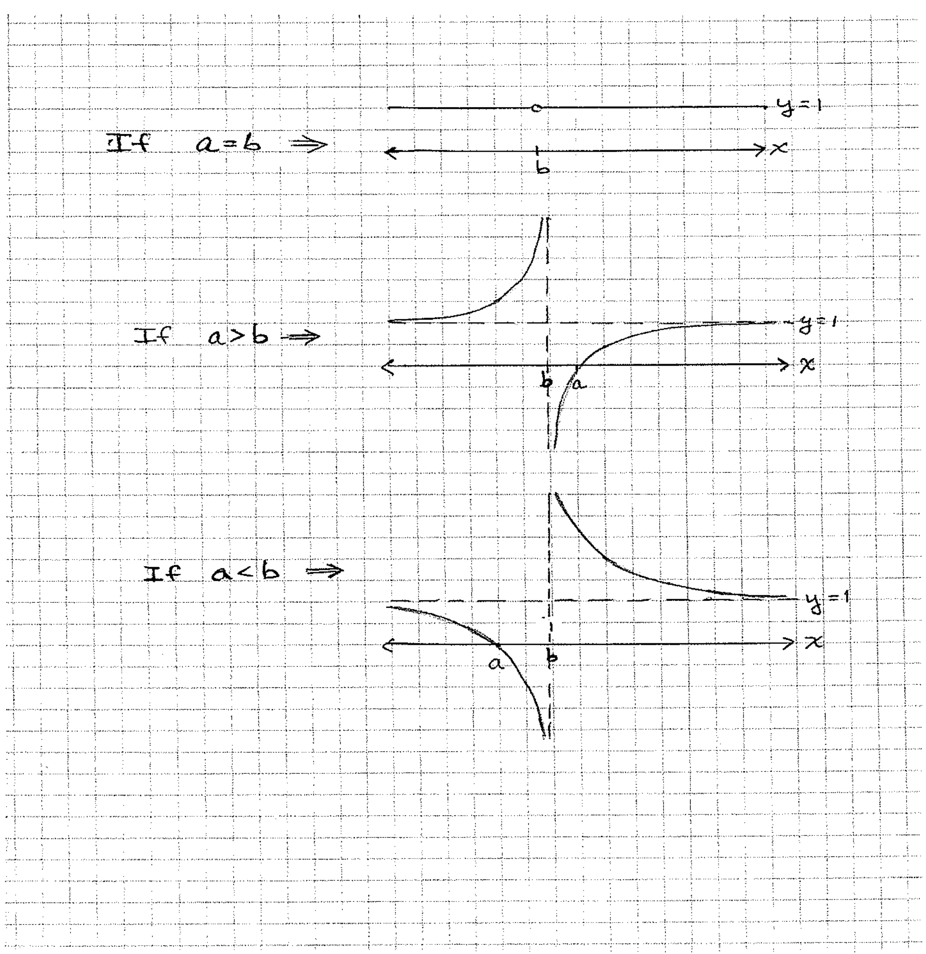


9. $f(x)=\dfrac{x-a}{x-b}$ for constants a and b

Note: If $a=b$, then $f(x)=1$ with domain $x \neq b$. The graph is the horizontal line $y=1$ with the point $(b,\ 1)$ removed. (see below)

The table below assumes that $a \neq b$:

Domain $x \neq b$	y-intercept $\left(0,\ \frac{a}{b}\right)$	x- intercept $(a,\ 0)$
$f'(x)=\dfrac{-b+a}{(x-b)^2}$	$f'=0$ never (since $a \neq b$)	f' DNE at $x=b$
Incr. On domain if $a>b$ Nowhere if $a<b$	Decr. Nowhere if $a>b$ On domain if $a<b$	
Loc min none	Loc max none	
$f''(x)=\dfrac{-2(-b+a)}{(x-b)^3}$	$f''=0$ never (since $a \neq b$)	f'' DNE at $x=b$
Conc.ave UP $(-\infty,\ b)$ if $a>b$ $(b,\ \infty)$ if $a<b$	Concave DOWN $(b,\ \infty)$ if $a>b$ $(-\infty,\ b)$ if $a<b$	P.O.I. none
Horiz. Asymp. $y=1$	Vertical Asymp. $x=b$	



10. $f(x) = \dfrac{x-a}{x+b}$ for constants a and b

Note: If $a = -b$, then $f(x) = 1$ with domain $x \neq -b$. The graph is the horizontal line $y = 1$ with the point $(-b,\ 1)$ removed. (see below)

The table below assumes that $a \neq -b$:

Domain $x \neq -b$	y-intercept $\left(0,\ -\frac{a}{b}\right)$	x- intercept $(a,\ 0)$
$f'(x) = \dfrac{b+a}{(x+b)^2}$	$f' = 0$ never (since $a \neq -b$)	f' DNE at $x = -b$
Incr. On domain if $a > -b$ Nowhere if $a < -b$	Decr. Nowhere if $a > -b$ On domain if $a < -b$	
Loc min none	Loc max none	
$f''(x) = \dfrac{-2(b+a)}{(x+b)^3}$	$f'' = 0$ never (since $a \neq -b$)	f'' DNE at $x = -b$
Conc.ave UP $(-\infty,\ b)$ if $a > -b$ $(b,\ \infty)$ if $a < -b$	Concave DOWN $(b,\ \infty)$ if $a > -b$ $(-\infty,\ b)$ if $a < -b$	P.O.I. none
Horiz. Asymp. $y = 1$	Vertical Asymp. $x = -b$