

## Quizzes for Math 404

**QUIZ 1.** a) Show that the set  $\{(x, y, z) : x - y + z = 0\}$  is a subspace of  $K^3$ .

b) Are the vectors  $(1, 1, 0)$ ,  $(1, 1, 1)$ ,  $(0, 1, -1)$  linearly independent? Justify your answer.

**QUIZ 2.** a) Show that

$$\text{span}(\{(1, 0, 1, 0), (1, 2, 2, 1), (0, 1, 0, 1), (0, 1, 1, 0)\}) = \text{span}(\{(1, 0, 1, 0), (0, 1, 0, 1), (0, 1, 1, 0)\}).$$

b) State the Going Up Theorem.

**QUIZ 3.** a) In a 7 dimensional vector space  $U$  is a subspace of dimension 3 and  $W$  is a subspace of dimension 5. Is it possible that  $U \cap W = \{0\}$ ? Justify your answer.

b) Find the reduced row-chelon form of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 3 & 2 & 3 & 2 \\ 2 & 2 & 1 & 2 \end{pmatrix}$$

Specify all the elementary operations using notation introduced in class.

**QUIZ 4.** A system of linear equations has augmented matrix

$$A = \begin{pmatrix} 2 & 4 & 1 & 1 & 4 \\ -1 & -2 & 0 & -1 & -1 \\ 2 & 4 & 3 & -1 & 5 \\ 1 & 2 & -1 & 1 & -1 \end{pmatrix}$$

- a) Write down this system of equations;
- b) Find the reduced row-echelon form of  $A$ ;
- c) What is the rank of  $A$ ?
- d) Solve the system of equations found in a).

**QUIZ 5.** Find a system of equations describing the subspace of  $\mathbb{R}^5$  spanned by the vectors  $(1, 1, -2, 0, 0)$ ,  $(1, 1, -1, -1, 0)$  and  $(1, 2, -1, 1, -3)$ .

**QUIZ 6.** a) Find a basis of  $\ker T$  and  $\text{Im}T$ , where  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  is given by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4, x_1 + x_3, x_1 - x_2 + x_3 - x_4, x_2 + x_4);$$

b) Is there a linear transformation  $T : \mathbb{R}^5 \rightarrow \mathbb{R}^5$  such that  $\ker T = \text{span}\{(1, 0, 0, 0, 0), (1, 1, 0, 0, 0), (1, 1, 1, 0, 0)\}$  and  $\text{Im}T = \text{span}\{(1, 0, 0, 0, 1), (0, 0, 1, 0, 0), (0, 1, 0, 1, 0)\}$ ?