

Problem 3 (due on Monday, October 10)

- a) Is there a function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $\left| \frac{f(x)+f(y)}{2} - f\left(\frac{x+y}{2}\right) \right| \leq \sin^2(x-y)$ for all $x, y \in \mathbb{R}$?
- b) Is there a function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $\left| \frac{f(x)+f(y)}{2} - f\left(\frac{x+y}{2}\right) \right| \leq \sin|x-y|$ for all $x, y \in \mathbb{R}$?

The answer to a) is positive, for example $f(x)=4x^2$ has the property. The answer to b) is negative. We received only one solution, from Prof. Vladislav Kargin, who solved a) and b) under additional assumption about f (essentially that f is continuous). For a detailed solution and some related material see the following link [Solution](#).

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