The Algebra Seminar

The seminar will usually meet in-person on Tuesdays in room WH-100E at 2:50 p.m. There should be refreshments served at 4:00 in room WH-102. Masks are optional.

Anyone wishing to give a talk in the Algebra Seminar this semester is requested to contact the organizers at least one week ahead of time, to provide a title and abstract. If a speaker prefers to give a zoom talk, the organizers will need to be notified at least one week ahead of time, and the following link will be used.

If needed, the following link would be used for a zoom meeting (Meeting ID: 981 8719 2351) of the Algebra Seminar:

Algebra Seminar Zoom Meeting Link

Organizers: Alex Feingold, Daniel Studenmund and Hung Tong-Viet

To receive announcements of seminar talks by email, please contact the organizers and explain your background and why you wish to subscribe. The usual subscription method has led to many spam requests.

Spring 2023

- **January 17**
  Organizational Meeting

  Please think about giving a talk in the Algebra Seminar, or inviting an outside speaker.

- **January 24**
  Alex Feingold (Binghamton University)
  Representations of Kac-Moody Lie Algebras (Part 1)

  Abstract: We will first review basic definitions and examples of finite dimensional Lie algebras and their representations. The infinite dimensional Heisenberg Lie algebra will be shown to have a representation on the space of polynomials in infinitely many variables. That space, V, is $\bf Z$-graded into subspaces V_n of total degree n with $\dim(V_n) = p(n)$, the classical partition function. Then we will give a brief introduction to the infinite dimensional Kac-Moody (KM) Lie algebras defined by generators and relations from a generalized Cartan matrix, A. We will discuss in detail the cases when A is 2×2, which give either affine or hyperbolic KM algebras. The affine case is related to the Heisenberg algebra, and we will present the root diagram and a weight diagram for an irreducible representation. We will discuss one hyperbolic example, Fib, and study its root system, and some irreducible representations (highest weight and non-standard).

- **January 31**
  Alex Feingold (Binghamton University)

Representations of Kac-Moody Lie Algebras (Continued)

- **February 7**
  Chris Schroeder (Binghamton University)

Understanding invariants of finite groups

Abstract: In this talk, we will consider the properties of finite groups that are preserved under isomorphism, which we call group invariants. The richness of finite group theory is reflected in the zoo of invariants that have been defined and studied. These invariants typically stem from the abstract group structure or the representation theory. We will discuss how group invariants can shed light on the classification of finite simple groups. This talk aims to be accessible and appealing to a broad mathematical audience.

- **February 14**
  Daniel Studenmund (Binghamton University)

Lie groups and pro-p groups

Abstract: We will discuss some results on pro-p groups, with a focus on cases in which pro-p groups behave like Lie groups in some sense. In particular, we will discuss the notion of an analytic structure on certain p-pro groups, and ways that this structure is analogous to the analytic structure on Lie groups. This talk will cover no original work, but rather focus on presenting work of Lazard, Serre, and others.

- **February 21**
  Robert Bieri (Binghamton University)

Groups of tile-permutations defined by finitary rearrangements of tessellations

Abstract: Let M be the Euclidean or hyperbolic n-space with a regular tessellation. By a finitary rearrangement of M (with respect to decompositions of M into finitely many pieces of a pre-specified shape) we mean the process of cutting M along tile-boundaries into finitely many tessellated pieces X_i each of which is either a single tile or isometric to one of the specified infinite shapes, and then mapping each piece by a tessellation-respecting isometric embedding $f_i : X_i \to M$, to a new position, with the property that the images $f_i(X_i)$ cover M and have pairwise disjoint interiors. The union of the maps f_i is well defined on all interior points of the tiles and induces a permutation of the set Ω of all tile-centers which we call a finitary piecewise isometric tile-permutation of M.

It is interesting to replace M by the abstract planar tree T with all its vertices of degree 3, view it as tessellated by its edges, and consider a corresponding construction. For if one restricts attention to finitary rearrangements of T with respect to decompositions of T whose infinite pieces specified to be rooted dyadic trees (with root of degree 2 and all other vertices of degree 3), then one recovers Richard Thompson's key tool to the structure of his groups: It exhibits the the elements of Thompson's group – up to finite permutations – as finitary piecewise planar-tree-isometric edge-permutations of T.

Moreover, the tree T occurs as the dual of the tessellation of the hyperbolic plane H^2 given by an ideal triangle and its iterated reflections over the sides; and the close connection between the isometries of H^2 and the planar-tree-isomorphisms of T allows one to show that Thompson's groups can also be interpreted as groups of piecewise hyperbolic-isometric tile-permutations of H^2.

The observation that Thompson's groups are part of the game could nurture hopes that other basic regular tessellations might lead to similar gem-stones. The Euclidean space E^n with its standard unit-cube tessellation is certainly the most natural down-to-earth case; and it is also natural to handle it by decomposing E^n into finitely many orthants: i.e. intersections of tessellated half-spaces. In this situation it is convenient to describe the set of all tile centers directly, identifying them with the lattice $\Omega = Z^n$. The decomposition of E^n into orthants thus decomposes Z^n into a finite pairwise disjoint union of single points, discrete rays, discrete quadrants, … , discrete rank-n orthants; and the elements of our piecewise Euclidean-isometric permutation group $G = pei(Z^n)$ have to rearrange them to a new disjoint-union-decomposition of Z^n. On the face of it this looks rather complicated, but those who nurture hope for gem-stones should be prepared to dig deep.

However, it turned out to be doable. In my talk I will indicate how a G-equivariant germs-of-orthants structure at infinity of Z^n with a corank-1 flow helps to get information on the group structure of G to show:

(1) The group of all piecewise Euclidean isometric permutations of Z^n is the fundamental group of a CW-complex with finite $(2n – 1)$-skeleton. This is joint work with Heike Sach, ArXiv:2016. (Recall that Thompson's groups are of type F_∞).

(2) For each $0\leq k\leq n$ the subgroup G_k consisting of all elements of G supported on the k-skeleton of E^n is normal in G. In my pandemic related sabbatical from BU I added: Every normal subgroup N of rank k in G_k contains the unique subgroup of index 2 of G_{k-1}. See J. London Math. Soc. 2022. (Recall that Thompson's groups V and T are simple and every normal subgroup of F contains F').

- **February 28**
  Banafsheh Akbari (Cornell University)

The Structure of Finite Groups affected by Vertex Neighborhoods of their Solubility Graphs

Abstract: The solubility graph associated with a finite group G is a simple graph whose vertices are the elements of G, and there is an edge between two distinct vertices if and only if they generate a soluble subgroup. Properties of this graph can have dramatic consequences on the structure of G. For example, it has recently been proved that if some non-identity element has prime degree then G is an

abelian simple group. So in this talk, we focus our attention on the set of neighbors of a vertex \$x\$ which we call it the solubilizer of \$x\$ in \$G\$, \$Sol_G(x)\$. We investigate both arithmetic and structural properties of this set and discuss how restrictions on the structure of this set affect the structure of \$G\$.

- **March 7**
  Alex Feingold (Binghamton University)

Fusion Rules for Affine Kac-Moody Algebras

Abstract: This is an expository introduction to fusion algebras for affine Kac-Moody algebras, with major focus on the algorithmic aspects of their computation and the relationship with tensor product decompositions. Explicit examples are included illustrating the rank 2 cases. Previous work of the author and collaborators on a different approach to fusion rules from elementary group theory is also explained.

- **March 14**
  Lucia Morotti (Heinrich-Heine-Universit&aumlt D&uumlsseldorf)

Decomposition matrices of spin representations of symmetric groups

Abstract: Not much is known about decomposition numbers of spin representations of symmetric groups. For example it is not even known whether in general the decomposition matrices are triangular. I will show how certain parts of the decomposition matrices look like and in particular focus on rows corresponding to spin representations labeled by partitions with at most 2 parts.

- **March 21**
  (? University)

Title

Abstract: Text of Abstract

- **March 28**
  (? University)

Title

Abstract: Text of Abstract

- **April 4**
  No Seminar (Spring Break)

- **April 11**
  Duc-Khanh Nguyen (SUNY Albany)

A generalization of the Murnaghan-Nakayama rule for K-k-Schur and k-Schur functions

Abstract: We introduce a generalization of K-k-Schur functions and k-Schur functions via the Pieri rule. Then we obtain the Murnaghan-Nakayama rule for the generalized functions. The rule are described explicitly in the cases of K-k-Schur functions and k-Schur functions, with concrete descriptions and algorithms for coefficients. Our work recovers the result of Bandlow, Schilling, and Zabrocki for k-Schur functions, and explains it as a degeneration of the rule for K-k-Schur functions. In particular, many other special cases promise to be detailed in the future.

- **April 18**
  Nicholas Packauskas (SUNY Cortland)

Growth of Betti Sequences over Commutative Rings

Abstract: One method of studying a local commutative ring is to study its category of finitely generated modules. An invariant of particular interest is a module's Betti sequence. This sequence contains various information about the module, but in particular it measures the growth of the minimal free resolution of the module. Studying such sequences can also provide information about the ring. In particular, it is known that if every finitely generated module has a minimal free resolution that grows on the order of a polynomial, then the ring must be a complete intersection. One can say much more in this case; each Betti sequence will be governed by a quasipolynomial of period 2. We will explore such sequences, and establish a bound on the discrepancy between the polynomials which govern their even and odd terms. The bound is computed using an invariant of the ring called its quadratic codimension. (Joint work with Lucho Avramov and Mark Walker.)

- **April 25**
  No Seminar

- **May 2**
  No Seminar (Friday Classes Meet)

- Pre-2014 semesters
- Fall 2014
- Spring 2015
- Fall 2015
- Spring 2016
- Fall 2016

- Spring 2017
- Fall 2017
- Spring 2018
- Fall 2018
- Spring 2019
- Fall 2019
- Spring 2020
- Fall 2020
- Spring 2021
- Fall 2021
- Spring 2022
- Fall 2022