

HW 1 REMARKS

Prop 1.23: Prove that

$$(m - n) - (p - q) = (m + q) - (n + p).$$

Here's one proof (of many possible) that is valid and which I expected your proofs to look something like. I tried to put thorough explanations for your benefit, which I did not expect you to do (you just had to put something like "(D)" if you use the distributive property for instance and you did not have to explain like I did below).

$$\begin{aligned}(m - n) - (p - q) &= (m - n) + \left(- (p + (-q)) \right) \quad [\text{def. of subtraction, } a - b = a + (-b) \text{ where } a = m - n \text{ and } b = p - q] \\ &= (m - n) + (-1)(p + (-q)) \quad [\text{since } \forall m \in \mathbb{Z}, -m = (-1)m; \text{ here, } m = p + (-q)] \\ &= (m - n) + \left((-1)p + (-1)(-q) \right) \quad [\text{distributive axiom}] \\ &= (m - n) + \left((-p) + (-(-q)) \right) \quad [\text{since } \forall m \in \mathbb{Z}, (-1)m = -m] \\ &= (m - n) + \left((-p) + q \right) \quad [\text{since } \forall m \in \mathbb{Z}, -(-m) = m] \\ &= (m + (-n)) + \left((-p) + q \right) \quad [\text{def. of subtraction, } m - n = m + (-n)] \\ &= m + \left((-n) + \left((-p) + q \right) \right) \quad [\text{associative axiom, } (a + b) + c = a + (b + c) \text{ where } a = m, b = -n, c = -p + q] \\ &= m + \left(\left((-n) + (-p) \right) + q \right) \quad [\text{associative axiom, } a + (b + c) = (a + b) + c \text{ where } a = -n, b = -p, c = q] \\ &= m + \left(q + \left((-n) + (-p) \right) \right) \quad [\text{commutative axiom, } a + b = b + a \text{ where } a = (-n) + (-p) \text{ and } b = q] \\ &= (m + q) + \left((-n) + (-p) \right) \quad [\text{associative axiom, } a + (b + c) = (a + b) + c \text{ where } a = m, b = q, c = (-n) + (-p)] \\ &= (m + q) + \left((-1)n + (-1)p \right) \quad [\text{since } \forall m \in \mathbb{Z}, -m = (-1)m] \\ &= (m + q) + (-1)(n + p) \quad [\text{distributive axiom}] \\ &= (m + q) + \left(- (n + p) \right) \quad [\text{since } \forall m \in \mathbb{Z}, -m = (-1)m] \\ &= (m + q) - (n + p) \quad [\text{def. of subtraction}]\end{aligned}$$