HOMEWORK 3: DUE FRIDAY MAR 6-TH.

1) Prove that for any two sets A and B,
$$A \setminus B = A \cap B^c$$
(assume x is in the first set, then prove it's in the second set and do the reverse).

2) Let A be a set and let $\{A_\alpha\}_{\alpha \in I}$ be a family of sets. Prove one of the following two:
$$A \cap \Big(\bigcup_{\alpha \in I} A_\alpha \Big) = \bigcup_{\alpha \in I} (A \cap A_\alpha)$$
or
$$A \setminus \Big(\bigcap_{\alpha \in I} A_\alpha \Big) = \bigcup_{\alpha \in I} (A \setminus A_\alpha).$$

3) Let $f : X \to Y$ be a function between two sets, X and Y, let $B, C \subseteq Y$, and let $\{A_\alpha\}_{\alpha \in I}$ be a family of subsets of X. Prove one of the following two:
$$f^{-1}(B \setminus C) = f^{-1}(B) \setminus f^{-1}(C)$$
or
$$f\Big(\bigcup_{\alpha \in I} A_\alpha \Big) = \bigcup_{\alpha \in I} f(A_\alpha).$$
NB: For the first equality you need the definition
$$\text{For } D \subseteq Y,\ x \in f^{-1}(D) \iff f(x) \in D.$$
For the second equality you need the definition
$$\text{For } A \subseteq X,\ y \in f(A) \iff y = f(x) \text{ for some } x \in A.$$

4) Prove Proposition 5.8 (i) or (ii) (not both) on page 35/36 of book.

5) Prove Proposition 5.17, page 37.

NOTE: I also asked you to prove some other problems (e.g. that $(A \setminus B) \setminus C = (A \setminus B) \setminus (B \setminus C)$ assuming some equalities of sets). However, please DO NOT do this problem. After talking with a student I see that there might be some unnecessary confusion on the other problems I assigned.