HOMEWORK 5: DUE FRIDAY MAY 8. DO TWO OF THE FOLLOWING THREE.

1) If A is an infinite set and B is a countable set, prove that $A \cup B$ and A have the same cardinality. (Prove this in a similar way that we proved $[0,1]$ and $(0,1)$ have the same cardinality, which was a similar proof that $\mathbb{R} = \mathbb{Q}^c \cup \mathbb{Q}$ and $\mathbb{Q}^c$ have the same cardinality.) You need to use the lemma: If I is an infinite set, then I has a countably infinite subset.

2) Using the result in 1), prove that $(0, 1000]$ and $[0,1)$ have cardinality $\mathfrak{c}$. (Recall that we proved any open interval (a,b) has cardinality $\mathfrak{c}$.)

3) Let A be a nonempty set. We shall define a function $\beta : P(A) \to 2^A$. Let $B \in P(A)$; then we need to define $\beta(B) \in 2^A$, that is, we need to define a function

$$\beta(B) : A \to \{0,1\}.$$

To simplify notation a bit, let $f := \beta(B)$. We define $f : A \to \{0,1\}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x \in B, \\ 0 & \text{if } x \notin B. \end{cases}$$

This defines $\beta(B) : A \to \{0,1\}$ and hence the function $\beta : P(A) \to 2^A$. Show that β is a bijection.