**(REVISED) TEST 1: DUE MONDAY MARCH 23.**

TEST RULES: You are allowed to use only your personal class notes, your personal previous HWs, the pdf files HW1,HW2 on our 330 website, and the 330 course pack. Collaborating, the internet, other books, etc ... (basically any help whatsoever) are NOT allowed.

1) Given $m, n, p, q \in \mathbb{Z}$, prove that

$$(m-n)(p-q) = (mp+nq) - (mq+np).$$

NOTES:

a) You are allowed to assume only the axioms of $\mathbb{Z}$ found in Section 1.1, the various properties of $\mathbb{Z}$ found in Section 1.2, and the definition of subtraction.

b) *Your proof should be as careful as the proof on the 330 webpage concerning Remarks on HW1.* Here's an example of what I mean. Consider the expression

$$mp + nq - mq - np \qquad (*),$$

which is illegal for the first test because on this first test you should not omit parentheses! You will get some points deducted if you omit parentheses. A couple examples of expressions that would be acceptable in place of $(*)$ include

$$((mp+nq) - mq) - np$$

or

$$mp + ((nq - mq) - np).$$

2) Prove that for all natural numbers $n$,

$$\sum_{k=1}^{n}(2k-1) = n^2,$$

where for any $a \in \mathbb{Z}$, $a^2 := a \cdot a$. (It's common to write $\sum_{k=1}^{n}(2k-1) = n^2$ as $1+3+5+\cdots+(2n-1) = n^2$.)

NOTES:

a) You are allowed to assume only the axioms of $\mathbb{Z}$ found in Section 1.1, the various properties of $\mathbb{Z}$ found in Section 1.2, the definition of subtraction, the definition of summation, the axioms of $\mathbb{N}$ and induction.

b) *Your proof should be as careful as the proof on the 330 webpage concerning Remarks on HW2.*

3) Let $A$ and $B$ be sets. Prove that

$$A \subseteq B \quad \text{if and only if} \quad A = A \cap B.$$

Recall, "if and only if" means you have to prove both statements:

a) If $A \subseteq B$, then $A = A \cap B$,

and also

b) If $A = A \cap B$, then $A \subseteq B$.

NOTES: To prove a) proceed as follows: Assume $A \subseteq B$, then to show that $A = A \cap B$, I want you to let $x \in A$ and show that $x \in A \cap B$, then let $x \in A \cap B$ and show that $x \in A$. The way I want you to show b) is to assume $A = A \cap B$, then to show that $A \subseteq B$, let $x \in A$ and show that $x \in B$.

4) Let $a, b, c, d \in \mathbb{N}$. Prove that if $a < b$ and $c < d$, then $ac < bd$.

NOTES: You are allowed to assume only the axioms of $\mathbb{Z}$ found in Section 1.1, the various properties of $\mathbb{Z}$ found in Section 1.2, the definition of subtraction, the definition of "$<$", the axioms of $\mathbb{N}$, and the fact that the sum and the product of natural numbers are natural numbers.

(You are not allowed to use any propositions or theorems concerning "$<$" that are in the course pack.)