PAPERS SUMMARY

LUCAS SABALKA

1. Geodesics in the braid group on three strands

In *Group theory, statistics, and cryptography*, volume 360 of *Contemp. Math.*, pages 133-150. Amer. Math. Soc., Providence, RI, 2004.

This paper is a revised version of my undergraduate thesis, completed at the University of Nebraska-Lincoln under my advisors Susan Hermiller and John Meakin.

In this paper, I compute the geodesic growth series of the braid group on three strands, B_3, with respect to the standard Artin generating set $\{a, b\}$. The *geodesic growth series* of a group with respect to a generating set is a formal power series whose n^{th} coefficient is the number of geodesic words of length n. It turns out that for B_3, the geodesic growth series is rational, and in this paper I expicitly compute a rational closed form. In the process, I characterized necessary and sufficient conditions for what a geodesic representative of an element in B_3 must have.

2. Discrete Morse theory and graph braid groups

with Daniel Farley. *Algebr. Geom. Topol.*, 5:1075-1109, 2005. Available online at: http://www.maths.warwick.ac.uk/agt/AGTVol5/agt-5-44.abs.html .

This paper, cowritten with Daniel Farley, appeared in Algebraic and Geometric Topology (2005). In it, Farley and I provide an explicit general method for computing presentations of graph braid groups. We do so by applying discrete Morse theory to certain CW-complexes related to graph braid groups. We analyze in detail presentations for tree braid groups, including example computations. We also provide a number of corollaries including bounds on homological dimensions in many cases.

3. Embeddings of right-angled Artin groups into graph braid groups

Geom. Ded., to appear. Available online at: www.arXiv.org/math.GR/0506253 .

In a recent paper by Crisp and Wiest, it is proven that any graph braid group embeds into some right-angled Artin group, based on an embedding of corresponding Eilenberg-Maclane spaces. In this paper I prove that the opposite direction also holds: any right-angled Artin group embeds into some graph braid group. I do so by constructing embeddings for any given right-angled Artin group. As a result, I find a braid group on a planar graph using only two strands which contains a hyperbolic surface subgroup, answering a previously open question.

4. On the cohomology rings of tree braid groups

with Daniel Farley. Submitted, 2006. Available online at:
www.arxiv.org/math.GR/0602444 .

The paper uses discrete Morse theory to analyze the cohomology ring structure of tree braid groups, explicitly characterizing the cup product. We use these cohomology calculations to show that the only tree braid groups which are also right-angled Artin groups are braid groups with $n < 4$ strands or braid groups on *linear trees*, where a tree is linear if all vertices of degree 3 or more lie on an embedded arc.

5. Braid groups on graphs

Ph.D. dissertation, U. of Illinois at Urbana-Champaign, 2006. Available online at:
www.math.uiuc.edu/ sabalka/SabalkaDissertation.pdf .

This is my doctoral dissertation. It includes most results from my papers on graph braid groups. It is meant to be a more readable, compact, and polished compilation of my earlier work. New results from my dissertation are being compiled in the paper On rigidity and the isomorphism problem for tree braid groups, below.

6. On presentations of graph braid groups

with Daniel Farley. In preparation.

This paper begins with a discussion of discrete Morse theory applied to general graph braid groups. Then, we prove a presentation theorem for planar two strand graph braid groups, in which every relator is a commutator corresponding to disjoint regions in the planar graph. This proves, for instance, that the first homology group of a planar two strand graph braid group is free abelian.

7. On Rigidity and the Isomorphism Problem for tree braid groups

Preprint, 2006.

In this paper, I obtain a positive answer to a version of the isomporphism problem for four and strand tree braid groups: given two tree braid groups on four or five strands G and G', is it algorithmically decidable if G and G' are isomorphic? The approach I take is to use the cohomology ring structure of a given tree braid group $G = B_n T$ for $n = 4$ or 5 to reconstruct the original tree T (up to homeomorphism). This implies that n strand tree braid groups and trees up to homeomorphism are in bijective correspondence for $n = 4$ or 5, and that any tree braid group $B_n T$ for $n = 4$ or 5 strands determines all other tree braid groups on T.