

Homework #4 Solution Set

February 16, 2006

(2.1#2) Define the following matrices:

$$A := \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, B := \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, C := \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, D := \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, E := \begin{bmatrix} -5 \\ 3 \end{bmatrix}.$$

Compute the following sums and products:

(a) $A + 2B$

Answer:

$$A + 2B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + 2 \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 2+14 & 0-10 & -1+2 \\ 4+2 & -5-8 & 2-6 \end{bmatrix} = \begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix}.$$

(b) $3C - E$

Answer: The matrices C and E do not have the same size, so they may not be added together.

(c) CB

Answer:

$$CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 7+2 & -5-8 & 1-6 \\ -14+1 & 10-4 & -2-3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}.$$

(d) EB

Answer: This product is undefined: E has only one column, while B has 2 rows.

(2.1#6) Let

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}.$$

(a) Compute AB by definition.

Answer: We have that

$$A\vec{b}_1 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix}$$

and

$$A\vec{b}_2 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}.$$

Thus,

$$AB = [A\vec{b}_1 A\vec{b}_2] = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}.$$

(b) Compute AB using the row-column rule.

Answer: We have:

$$AB = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4-4 & 12+2 \\ -3+0 & -9+0 \\ 3+10 & 9-5 \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}.$$

(2.1#27) Let $\vec{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute the following products.

Answer:

(a)

$$\vec{u}^T \vec{v} = [-2 \quad 3 \quad 4] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -2a + 3b + 4c.$$

(b)

$$\vec{v}^T \vec{u} = [a \quad b \quad c] \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} = -2a + 3b + 4c.$$

(c)

$$\vec{u} \vec{v}^T = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix} [a \quad b \quad c] = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ 4a & 4b & 4c \end{bmatrix}.$$

(d)

$$\vec{v} \vec{u}^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} [-2 \quad 3 \quad 4] = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ 4a & 4b & 4c \end{bmatrix}.$$