Homework #5 Solution Set

March 2, 2006

(2.6#1) Construct the consumption matrix for the economy stated in your book, and determine what intermediate demands are created if agriculture plans to produce 100 units.

Answer: The consumption matrix is:

$$C = \begin{bmatrix} .10 & .60 & .60 \\ .30 & .20 & 0 \\ .30 & .10 & .10 \end{bmatrix},$$

where the first row and column refers to the manufacturing sector, the second to agriculture, and the third to services.

The intermediate demand created by agriculture producing 100 units is $100 \begin{bmatrix} .60 \\ .20 \\ .10 \end{bmatrix} = \begin{bmatrix} 60 \\ 20 \\ 10 \end{bmatrix}$.

(2.6#9) Solve the Leontief production equation for an economy with three sectors, given that

$$C = \begin{bmatrix} .2 & .2 & 0 \\ .3 & .1 & .3 \\ .1 & 0 & .2 \end{bmatrix}, \text{ and } \vec{d} = \begin{bmatrix} 40 \\ 60 \\ 80 \end{bmatrix}.$$

Answer: We simply solve the matrix equation $(I - C)\vec{x} = \vec{d}$. Row reducing the matrix $[(I - C)|\vec{d}]$, we eventually get the answer $\vec{x} = \begin{bmatrix} 82.8 \\ 131.0 \\ 110.3 \end{bmatrix}$.

(3.1#2) Compute the determinant of $A = \begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$ using a cofactor expansion across the first row, and then down the second column.

Answer: Across the first row, we have:

$$|A| = \begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix} = 0 - 5 \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & -3 \\ 2 & 4 \end{vmatrix} = -5 * 4 + 22 = 2.$$

Down the second column, we have:

$$|A| = \begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix} = -5 \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + (-3) \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 4 \begin{vmatrix} 0 & 1 \\ 4 & 0 \end{vmatrix} = -5 * 4 + (-3) * (-2) - 4 * (-4) = 2.$$

(3.1#9) Compute the determinant of the following matrix by cofactor expansions using the least amount of computation possible.

$$A = \begin{bmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{bmatrix}$$

Answer: We begin by using cofactor expansion across the third row:

$$|A| = \begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix} = 2 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix}.$$

In the resulting matrix, we use cofactor expansion across the first row:

$$2 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix} = 2 * 5 \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = 2 * 5 * (7 - 6) = 10.$$