Homework #6 Solution Set

March 16, 2006

(4.1#2) Let W be the union of the first and third quadrants in the xy-plane. That is, let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} | xy \geq 0 \right\}$.

(a) If $\vec{u}$ is in W and c is any scalar, is $c\vec{u}$ in W? Why?

Answer: Yes. Let $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. Then $c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$. Because $c^2 \geq 0$ for any real scalar c and $u_1 u_2 \geq 0$ since $\vec{u} \in W$, $(cu_1)(cu_2) = (c^2)(u_1 u_2) \geq 0$. But this means exactly that $c\vec{u} \in W$.

(b) Find specific vectors $\vec{u}$ and $\vec{v}$ in W such that $\vec{u}+\vec{v}$ is not in W. This shows that W is not a vector space.

Answer: Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$. Then $\vec{u}, \vec{v} \in W$, but $\vec{u} + \vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \notin W$.

(4.1#13) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, , $\vec{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and , $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

(a) Is $\vec{w}$ in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$? How many vectors are in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

Answer: No: the only 3 vectors in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.

(b) How many vectors are in $Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

Answer: There are infinitely many vectors in this span, as the span contains any possible linear combination of the three vectors.

(c) Is $\vec{w}$ in the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$? Why?

Answer: If $\vec{w} \in Span\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$, then $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{w}$ has a solution - i.e. the matrix $[\vec{v}_1\ \vec{v}_2\ \vec{v}_3|\vec{w}]$ corresponds to a consistent system of linear equations. Row reducing, we see that:

$$[\vec{v}_1\ \vec{v}_2\ \vec{v}_3|\vec{w}] \sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

which does correspond to a consistent system of linear equations. Thus, $\vec{w} \in W$.

(4.1#19) If a mass m is placed at the end of a spring, and if the mass is pulled downward and released, the mass-spring system will begin to oscillate. The displacement y of the mass from its resting position is given by a function of the form

$$y(t) = c_1 \cos \omega t + c_2 \sin \omega t$$

where ω is a constant that depends on the spring and the mass. Show that, for a fixed ω, the set of all functions H of this form with c_1 and c_2 arbitrary is a vector space.

Answer: As, for any c_1 and c_2, $y(t)$ is a function and the space of all functions is a vector space, it suffices to check that H is a subspace.

(a) $\vec{0} \in H$: Simply set $c_1 = c_2 = 0$.

(b) If $(c_1 \cos \omega t + c_2 \sin \omega t) \in H$ and $(d_1 \cos \omega t + d_2 \sin \omega t) \in H$ then $((c_1 \cos \omega t + c_2 \sin \omega t) + (d_1 \cos \omega t + d_2 \sin \omega t)) \in H$:

$$(c_1 \cos \omega t + c_2 \sin \omega t) + (d_1 \cos \omega t + d_2 \sin \omega t) = ((c_1 + d_1) \cos \omega t + (c_2 + d_2) \sin \omega t) \in H.$$

(c) If $(c_1 \cos \omega t + c_2 \sin \omega t) \in H$ and $c \in \mathbb{R}$ then $c(c_1 \cos \omega t + c_2 \sin \omega t) \in H$:

$$c(c_1 \cos \omega t + c_2 \sin \omega t) = ((cc_1) \cos \omega t + (cc_2) \sin \omega t) \in H.$$

(4.1#22) Let F be a fixed 3×2 matrix, and let H be the set of all marixces A in $M_{2\times 4}$ with the property that $FA = 0$. Determine if H is a subspace of $M_{2\times 4}$.

Answer: The subset H of $M_{2\times 4}$ is a subspace. For, we check the three properties that H needs to be a subspace:

(a) $\vec{0} \in H$: We have that $F0 = 0$, so $0 \in H$.

(b) If $A \in H$ and $B \in H$ then $(A + B) \in H$: If $FA = 0$ and $FB = 0$, then $F(A + B) = FA + FB = 0 + 0 = 0$, so $(A + B) \in H$.

(c) If $A \in H$ and $c \in \mathbb{R}$ then $cA \in H$: If $FA = 0$, then $F(cA) = c(FA) = c0 = 0$, so $cA \in H$.