Homework #9 Solution Set

April 27, 2006

(6.1#2) Compute the following:

Answer:

- $w \cdot w = 9 + 1 + 25 = 35.$
- $x \cdot w = 18 + 2 - 15 = 5.$
- $(x \cdot w)/(w \cdot w) = 5/35 = 1/7.$

(6.2#10) Show $\left\{ \vec{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$ is an orthogonal basis for $\mathbb{R}^3$ and express $\vec{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of them.

Answer: To see that we have an orthogonal basis, compute dot products: $\vec{u}_1 \cdot \vec{u}_2 = 6 - 6 + 0 = 0$, $\vec{u}_1 \cdot \vec{u}_3 = 3 - 3 + 0 = 0$, $\vec{u}_2 \cdot \vec{u}_3 = 2 + 2 - 4 = 0$.

To express $\vec{x}$ as a linear combination, project:

$$\vec{x} = proj_{Span\{\vec{u}_1, \vec{u}_2 \vec{u}_3\}} \vec{x} = (4/3)\vec{u}_1 + (1/3)\vec{u}_2 + (1/3)\vec{u}_3.$$

(6.2#15) Compute the distance from $\vec{y} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ to $Span\{\vec{u} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}\}$.

Answer: We have that $dist(\vec{y}, Span\{\vec{u}\}) = ||\vec{y} - proj_{\vec{u}}\vec{y}||$, where $proj_{\vec{u}}\vec{y} = (3 * 8 + 1 * 6)/(8^2 + 6^2)\vec{u} = (3/10)\vec{u}$. Thus,

$$dist(\vec{y}, Span\{\vec{u}\}) = || \begin{bmatrix} 3 - 2.4 \\ 1 - 1.8 \end{bmatrix} || = || \begin{bmatrix} .6 \\ -.8 \end{bmatrix} || = 1.$$

(6.3#1) Let $\vec{u}_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}, \vec{u}_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}$. Write $\vec{x} = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$ as a sum of two vectors, one in $Span\{\vec{u}_4\}$ and one in $Span\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$.

Answer: As the $\vec{u}_i$ are pairwise orthogonal, the orthogonal component $\vec{z}$ of $\vec{x}$ with respect to $\vec{u}_4$ is in $Span\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$. We have:

$$proj_{\vec{u}_4}\vec{x} = (50 + 24 - 2 + 0)/(5^2 + 3^2 + 1^2 + 1^2)\vec{u}_4 = 2\vec{u}_4 \in Span\{\vec{u}_4\}$$

and

$$\vec{z} = \vec{x} - proj_{\vec{u}_4}\vec{x} = \begin{bmatrix} 0 \\ -2 \\ 4 \\ -2 \end{bmatrix} \in Span\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}.$$

We are done, since

$$\vec{x} = proj_{\vec{u}_4}\vec{x} + \vec{z}.$$

(6.3#19) *I am afraid I copied this problem incorrectly when I was typing it, and I did not bring my book with me when travelling, so I cannot give the exact answer - only how to do it.*

It can be shown that $\vec{u}_3$ is not in the subspace W spanned by $\vec{u}_1$ and $\vec{u}_2$. Use this fact to construct a nonzero vector $\vec{v}$ in $\mathbb{R}^3$ that is orthogonal to $\vec{u}_1$ and $\vec{u}_2$.

Answer: Since $\vec{u}_3$ is not in W, the orthogonal component of $\vec{u}_3$ with respect to W is nonzero, and by definition is orthogonal to W and hence to $\vec{u}_1$ and $\vec{u}_2$. Thus, $\vec{u}_3 - proj_W \vec{u}_3$ is as desired.