Math 225 Introductory Matrix Theory
Review Sheet

1 Linear Equations

- Definitions: linear equation, system of linear equations, solution, solution set, row equivalent, augmented matrix, pivot position, pivot column, column vector, vector equation, homogeneous,
- Elementary row operations
- Row Echelon Form and Reduced Row Echelon Form; the Row Reduction Algorithm
- General solutions of matrix equations: parametric descriptions and parametric vector descriptions
- Linear combinations and Span; linear independence
- Correspondence between systems of linear equations, vector equations, and matrix equations
- Applications of linear systems: economics, network flow, chemistry

2 Matrix Algebra

- Definitions: diagonal entries, main diagonal, diagonal matrix, transpose
- Basic matrix operations: addition, scalar multiplication, matrix multiplication
- Inverses: the 2×2 formula and the general method: $[A|I] \rightarrow [I|A^{-1}]$; properties
- The Invertible Matrix Theorem (see also Section 4.6)
- Applications: economics - $(I - C)\vec{x} = \vec{d}$

3 Determinant

- Cofactors
- Definition of determinant using cofactor expansion; simplified computation for triangular matrices
- How determinant changes with row operations; how to compute determinant using Row Reduction Algorithm

- A is invertible if and only if $det\ A \neq 0$
- $det\ A^T = det\ A$, $det\ AB = (det\ A)(det\ B)$
- $A^{-1} = \frac{1}{det\ A} adj\ A$
- Relationship between determinant and volume

sectionVector Spaces

- Examples of vector spaces: $\mathbb{R}^n$, $\mathbb{P}_n$, $\mathbb{P}$, $\{\vec{0}\}$, etc.
- Definition of subspace (3 properties)
- Linear independence, spanning sets, and bases
- A span is always a subspace
- Null Space, Column Space, Row Space: How to compute them and a basis for each
- Dimension; $rank = dimCol\ A = dimRow\ A = n - dimNul\ A$ for A $m \times n$

4 Eigenvectors and Eigenvalues

- Definitions: eigenvector, eigenvalue, eigenspace, characteristic equation, characteristic polynomial, multiplicity of eigenvalues
- Eigenvalues of a triangular matrix
- A is invertible if and only if 0 is not an eigenvalue
- Distinct eigenvalues implies linearly independent eigenvectors
- Similar matrices: definition, effect on characteristic polynomial
- Diagonalization: need n linearly independent eigenvectors - then you can form P and D so $A = PDP^{-1}$

5 Orthogonality and Inner Product

- Definitions: inner/dot product, lenth, unit vector, distance, orthogonal, orthogonal complement $W^{\perp}$, orthogonal set, orthogonal basis
- $(Row\ A)^{\perp} = Nul\ A$, $(Col\ A)^{\perp} = Nul\ A^T$
- Orthogonality implies linear independence
- Orthogonal projection:

$$\hat{y} = proj_{Span\{\vec{u_1},\ldots,\vec{u_k}\}}\vec{y} = \frac{\vec{y} \cdot \vec{u_1}}{\vec{u_1} \cdot \vec{u_1}}\vec{u_1} + \ldots + \frac{\vec{y} \cdot \vec{u_k}}{\vec{u_k} \cdot \vec{u_k}}\vec{u_k}$$

- Orthogonal component $\vec{z} = \vec{y} - \hat{y} = \vec{y} - proj_W \vec{y}$
- Orthonormal set/basis - simplifies orthogonal projection
- An $m \times n$ matrix U has orthonormal columns if and only if $U^T U = I$; if $U^{-1} = U^T$, U is an *orthogonal* matrix
- Least squares problem/solutions: $A^T A\vec{x} = A^T\vec{b}$
- Section 6.6 will not be on the test.