

# 1 Math 375 Final Practice Questions

1. Find all solutions to  $z^6 = -9$ .
2. Show that  $|z + w|^2 - |z - w|^2 = 4 \operatorname{Re}(z\bar{w})$  for any  $z, w \in \mathbb{C}$ .
3. Show that if  $f$  and  $\bar{f}$  are both holomorphic in a region  $G$ , then  $f$  is constant in  $G$ .
4. State the Cauchy-Riemann equations.
5. Find the Möbius transformations satisfying each of the following. Write your answers in standard form:
  - (a)  $0 \mapsto 0, 1 \mapsto 1, \infty \mapsto 2$
  - (b)  $i \mapsto -1, 2i \mapsto -2, 0 \mapsto 0$ .
6. Find the fixed points in  $\mathbb{C}$  of  $f(z) = \frac{z^2}{2z+i}$ .
7. Find all solutions to the equation  $\exp(z) = \pi i$ .
8. Find the length of the curve  $\gamma$  parametrized by  $\gamma(t) = e^{it} + i\pi e^{it}$  for  $0 \leq t \leq \pi$ .
9. Compute  $\int_{\gamma} z^2 dz$ , where  $\gamma$  is the semicircle from 1 through  $i$  to  $-1$ .
10. Prove Liouville's Theorem: Every bounded entire function is constant.
11. State the First Fundamental Theorem of Calculus.
12. Show that  $e^x \sin y$  is harmonic on  $\mathbb{C}$ .
13. Prove that  $\mathbb{Z}$  is complete.
14. Derive a formula for the product of two power series.
15. State the Maximum-Modulus Theorem.
16. Find the Taylor series about 0 for the following functions.
  - (a)  $(z^2 - 1)e^z$
  - (b)  $\frac{1}{1+z}$
  - (c)  $\frac{1}{e^z}$
17. Find the multiplicities of all zeros of  $(1 + z^2)^3$ .
18. Prove that if  $f$  is entire and constant on the disk  $D_1(0)$  then  $f$  is constant.
19. Let  $\gamma$  be the circle of radius 3 centered at 0. Compute the following integrals.
  - (a)  $\int_{\gamma} \frac{1}{z^2+1} dz$
  - (b)  $\int_{\gamma} \frac{\sin z}{z^2} dz$
20. Find the poles of  $\frac{z}{1-e^z}$  and determine their orders.