# Homework Assignment for MATH 505 Analysis I

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**Homework 1:** (Due to February 6)

Problem 10, 14, 15, 16, 23 in exercises for Chapter 1.

**Homework 2:** (Due to February 13)

Problem 2, 3, 4, 9, 18 in exercises for Chapter 1.

**Optional:** Problem 1, 5, 20, 21.

**Homework 3:** (Due to February 20)

Problem 11, 13, 19, in exercises for Chapter 1.

**Problem A:**If $f_n : X \to [0, \infty)$ is measurable on a measure space $(X, \Sigma, \mu)$, for $n = 1, 2, \cdots$, and $f(x) = \sum_{n=1}^{\infty} f_n(x)$ for $x \in X$, prove that

$$\int_X f(x) d\mu = \sum_{n=1}^{\infty} \int_X f_n(x) d\mu$$

(Hint: use monotone convergence Theorem)

**Problem B:** Let $\{E_n\}_{n=1}^{\infty}$ be a family of subsets in a measure space $(X, \Sigma, \mu)$, and $\sum_{n=1}^{\infty} \mu(E_n) < \infty$, prove that almost all $x \in X$ lie in at most finite many subsets of $\{E_n\}_{n=1}^{\infty}$. (Hint: Use Problem A)

**Optional:** Problem 6, 7, 8, 22.

**Homework 4:** (Due to February 27)

Problem 24, 26, 28, in exercises for Chapter 1.

Let $f_n : X \to [0, \infty)$ be measurable functions on $(X, \Sigma, \mu)$. We say $f_n \to f$ in measure if $\mu(E_{n,\epsilon}) \to 0$ as $n \to \infty$, for each $\epsilon > 0$, where

$$E_{n,\epsilon} = \{x \in X | \, |f_n(x) - f(x)| > \epsilon\}.$$

**Problem A:** Show that, if $f_n \to f$ in measure, then there are some subsequence $f_{n_j}(x) \to f(x)$ for almost every $x \in X$.

**Problem B:** Show that the Dominated Convergence Theorem continues to hold if one replaces $f_n(x) \to f(x)$ for almost every $x \in X$ by $f_n \to f$ in measure.

**Optional:** Problem 25, 27.

**Homework 5:** (Due to March 6)

Problem 1, 2, in exercises for Chapter 2.

**Problem A:** Let $K$ be an open convex set and $f : K \to \mathbf{R}$ be a convex function, show that $f$ is a continuous function on $K$.

**Problem B:** Let $K \subset \mathbf{R}$ be an open interval and $f : K \to \mathbf{R}$ be a convex function, show that there exists at least one support line of the graph of $f$ at each $x \in K$. And if $f$ is strictly convex at some point $x_0 \in K$, show that

$$f(y) > f(x_0) + V(y - x_0)$$

for $y \neq x_0$ and $V$ is the constant from support line.

**Problem C:** Let $K \subset \mathbf{R}$ be an open interval and $f : K \to \mathbf{R}$ be a convex function, show that $f$ always has a **right derivative** $f'_-(x)$ and **left derivative** $f'_+(x)$ for each $x \in K$.

**Optional:** Problem 18.

**Homework 6:** (Due to March 13)

Problem 4, 5, 6, 9, 19, in exercises for Chapter 2.

**Optional:** Problem 3, 21, 22.

**Homework 7:** (Due to March 20)

Problem 7,11,13,15,16, in exercises for Chapter 2.

**Optional:** Problem 8,10, 12, 14, 17.

**Homework 8:** (Due to April 3)

Problem 1, 2, 4, 6, 9, in exercises for Chapter 3.

**Optional:** Problem 3, 5

**Homework 9:** (Due to April 22)

Problem 1, 2, 3, 4, in exercises for Chapter 4.

**Homework 10:** (Due to May 1)

Problem 2, 3, 4, 6, 9 in exercises for Chapter 5.